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SIMPLIFIED SOLUTIONS TO THE INTERIOR BALLISTIC PROBLEMS OF CARTRIDGE ACTUATED DEVICES

by

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ABSTRACT

This report consists of the derivation of approximate analytic solutions to the interior ballistic equations of cartridge-actuated devices. A unique feature of the system is that linear burning rates need not be assumed. The solutions are presented in graphical form, thereby permitting the rapid solution to a variety of ballistic problems. Several sample problems are included. A complete list of the types of problems discussed is as follows:

A. Determination of maximum pressure and ejection velocity for given propellant loading.

1. Linear form function
2. Quadratic form function
3. Horizontal and nonhorizontal firing
4. With and without covolume correction

B. Determination of grain design parameters to yield given performance requirements.

1. Given maximum pressure and ejection velocity
2. Given maximum pressure
3. Given ejection velocity

C. Determination of ballistic effects of small changes in:

1. Mass accelerated
2. Impetus
3. Charge weight
4. Web size
5. Burning-rate coefficient
6. Chamber volume
7. Stroke length
8. Cross-sectional area of tube

D. Design of a scale model to reproduce ballistics of a larger cartridge-actuated device.

FOREWORD

This theoretical interior ballistic study was carried out under WEPTASK NO. RMMO 33 210/210-1/F008-11-001, "Power Sources for Actuated Devices (Miscellaneous); Research and Development", authorized by reference (a). This work was performed under Phase II of Contract No. N178-7503 by the Atlantic Research Corporation, under the technical direction of Naval Weapons Laboratory, Dahlgren, Virginia.

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INTRODUCTION

In experimental work with catapults and other cartridge-actuated devices, it is frequently desirable to be able to predict the maximum pressure and ejection velocity for a given loading with a minimum of computational effort. The interior ballistic performance of a cartridge-actuated device is governed by a system of non-linear differential equations which can only be solved by tedious, time-consuming numerical methods. Indeed, a single solution by such methods often requires as much as eight or ten hours using a desk calculator. Although many short methods have been developed for the determination of maximum pressure and ejection velocity, (b-g)[1] the assumptions upon which they are based, e. g., linear burning rates, neglected energy terms, etc., quite often lead to errors of considerable magnitude in cartridge-actuated device applications.

It is also desirable to be able to solve the companion problem, i. e., the charge weight and grain dimensions required to produce a given maximum pressure and ejection velocity. As far as is known, until this time no non-empirical method has been developed for solving this problem directly, it being necessary to employ a nonconverging trial-and-error process.

The purpose of this report is to provide a simple but accurate solution to both types of problems. A typical calculation using the methods developed here can be completed in about 15 minutes and requires nothing more than training in high-school mathematics. Results appear to agree to within 5 or 6 per cent of those obtained through numerical integration, although an exhaustive comparison of results has not been made. Simplifying assumptions are introduced which permit an analytic solution to the ballistic equations. The results are presented as families of curves relating certain dimensionless variables. Numerical examples are then presented to illustrate the use of the curves in solving the types of problems.

[1]Letters in parentheses indicate references at end of report.

The nomenclature used here has been made to correspond as closely as possible to that employed in a previous Atlantic Research Corporation report published in 1958 (i). Particular attention should be given to the units assigned to the parameters. These are included along with the nomenclature in Appendix G.

For brevity, a cartridge actuated device is hereinafter referred to as a CAD.

I. FUNDAMENTAL EQUATIONS

The equations presented in this section are for the most part derived in the report (i) cited above. They are included here to emphasize changes in notation and to make this a self-contained report. The basic equations are divided into two groups, those applicable during propellant burning and those applicable after the propellant has been consumed (after "burnt").

A. During Burning

1. The Equation of Motion of the load equates the net force on the piston to the mass in motion times the acceleration, as follows:

$$p_s A - F_f - \frac{w}{g_c} g \sin\theta = \frac{w}{g_c} \frac{d^2x}{dt^2} \tag{1.1}$$

where p_s is the pressure on the base of the piston, A is the cross-sectional area of the piston, F_f is the total resisting force due to friction, w is the gross accelerated mass, g is the acceleration due to gravity, $\frac{1}{g_c}$ is the factor for converting pounds of mass to slugs, θ is the angle of launch measured from the horizontal, x is the distance traveled by the piston, and t is the time measured from the beginning of motion of the load.

2. The Equation of State is the relation between the pressure, density, and temperature of the propellant gases. The Abel equation which applies with sufficient accuracy for most CAD's may be written,

$$p\left[\frac{12\,Ax + U_i}{C} - \left(\eta - \frac{1}{\rho}\right)\right] = 12\,NRT \tag{1.2}$$

where p is the space-average gas pressure, U_i is the initial free volume available to the gases, C is the mass of propellant burned at time t, η is the covolume factor, ρ is the density of the solid propellant, N is the number of moles of gas per unit mass, R is the universal gas constant, and T is the space-average gas temperature. The factor 12 is introduced to correct for the inconsistent units employed here as in most interior ballistic systems.

3. The Energy Balance Equation, assuming no gas leakage, states that the total energy lost by the propellant gases is equal to the kinetic energy of the load in motion, plus the potential energy of the load, plus the kinetic energy of the propellant and propellant gases, plus the energy lost through heat transfer through the CAD walls, plus the energy dissipated in overcoming friction. Thus,

$$C_v C(T_v - T) = \frac{w}{2g_c}\left(\frac{dx}{dt}\right)^2 + \frac{wgx \sin\theta}{g_c} + E_p + E_h + \int_0^x F_f\, dx \tag{1.3}$$

where C_v is the average constant-volume specific heat of the propellant gases over the range of temperatures in the CAD, T_v is the constant volume flame temperature of the gases, E_p is the kinetic energy of the propellant and propellant gases, and E_h is the energy lost through heat transfer through the launcher wall.

4. The Burning Rate Equation of the propellant expresses the propellant burning rate as a function of the space-average pressure. The following equation fits the experimental data of most propellants.

$$r = Bp^n \tag{1.4}$$

where r is the propellant burning rate with dimensions $\frac{\text{distance}}{\text{time}}$, n is the burning-rate exponent with a usual range of $0.2 \leq n \leq 1.2$, and B is a constant of proportionality.

5. The final relation required is the Form Function of the propellant grain. When a solid propellant burns, all the uninhibited surface recedes at the same rate assuming simultaneous ignition of all surfaces. Thus, there

is a simple geometric relation called the form function which relates the mass of the propellant burned at any time to the distance burned through the grain at that time. Most propellant grains presently in use have a cubic form function of the type

$$C = \Lambda_1 L + \Lambda_2 L^2 + \Lambda_3 L^3 \tag{1.5}$$

where Λ_1, Λ_2, Λ_3 are constants depending upon the geometry of the grain, and L is the distance burned through the grain. (See Appendix A for a discussion of the form functions of some of the more common grain configurations.)

B. After Burnt

Let Z be any one of the variables. The value of Z at grain burnout will be denoted by Z_b. L_b will be referred to as the "web",[1] and C_b as the charge weight. After burnout, the ballistics are governed by equations (1.1), (1.2), and (1.3) where C is replaced by the constant C_b.

II. DEVELOPMENT OF THE EQUATIONS OF MOTION AND ENERGY BALANCE

A. Kinetic Energy of Propellant and Propellant Gases

The motion of the propellant and propellant gases is described by the usual partial differential equations of fluid flow. The solution, however, is entirely too cumbersome to be included in any interior ballistic system. An approximate solution developed by Kent[2] is used in most systems.

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[1] Some interior ballisticians define the web as the minimum distance between the initial surfaces of the grain. Using this definition, the web is usually equal to $2L_b$. Care should be taken so as not to confuse the two definitions.

[2] See reference (b), pp. 141-147 for the derivation.

According to this solution, the space-average pressure is related to the pressure on the base of the piston by the expression

$$p = \left(1 + \frac{C_b}{\sigma w}\right) p_s \tag{2.1}$$

where σ is a constant depending upon the ratio C_b/w, with a usual value of about 3. The kinetic energy of the propellant and propellant gases is related to that of the accelerated mass by

$$E_p = \frac{C_b}{\sigma w}\left[\frac{w}{2g_c}\left(\frac{dx}{dt}\right)^2\right] . \tag{2.2}$$

The value of the ratio $C_b/\sigma w$ for most CAD's is of the order of 10^{-4}. Hence, little inaccuracy is introduced by setting $p_s = p$ in equation (1.1), and $E_p = 0$ in equation (1.3).

B. Friction Energy Losses

The friction force, F_f, is assumed to be a constant percentage of the pressure on the base of the piston, p_s. (See reference (b), pp. 8-9 for justification of the assumption.) Let the constant of proportionality be $\frac{K_1 A}{1+K_1}$; then, since $p_s = p$, equation (1.1) becomes

$$pA = (1 + K_1)\ \frac{w}{g_c}\left(\frac{d^2x}{dt^2} + g \sin \theta\right) . \tag{2.3}$$

The assumption of proportionality also permits the evaluation of the integral $\int_0^x F_f\, dx$ in equation (1.3). Let $v = \frac{dx}{dt}$, then

$$\int_0^x F_f dx = K_1 \int_0^x \frac{pAdx}{1 + K_1} = \frac{K_1 w}{2g_c} (v^2 + 2\ gx \sin \theta) . \tag{2.4}$$

Setting $(1 + K_1)\ w = w_1$, an "effective mass" being accelerated, equations (1.1) and (1.3) simplify to

$$pA = \frac{w_1}{g_c}\left(\frac{dv}{dt} + g \sin \theta\right) \tag{2.5}$$

and

$$C_v C (T_v - T) = \frac{w_1}{2g_c} (v^2 + 2\, gx \sin\theta) + E_h \,. \tag{2.6}$$

The value of K_1 is usually obtained empirically from ejection-velocity data, and an integration of experimental pressure-time curves. It follows from equation (2.3) that

$$\int_0^{t_m} p\,dt = \frac{(1 + K_1)w}{Ag_c} (v_m + g\, t_m \sin\theta) \tag{2.7}$$

where subscript m, here and elsewhere, denotes values at the completion of the piston stroke. Rearranging equation (2.7), the value of K_1 is found to be

$$K_1 = \frac{Ag_c \int_0^{t_m} p\,dt}{w\,(v_m + g\, t_m \sin\theta)} - 1 \,. \tag{2.8}$$

C. Heat Energy Losses

It was shown in the previous Atlantic Research Corporation report (i), pp. 4-5, that the energy lost through heat transfer is approximately proportional to the kinetic energy of the accelerated mass, i. e.,

$$E_h \approx K_2\, (1/2)\, \frac{w}{g_c}\, v^2 \tag{2.9}$$

where a typical value of K_2 is 0.25. Substituting this expression for E_h into equation (2.6) yields the final form of the energy balance equation

$$C_v C(T_v - T) = \left(\frac{1 + K_1 + K_2}{1 + K_1} \right) \frac{w_1 v^2}{2g_c} + \frac{w_1}{g_c}\, gx \sin\theta \,. \tag{2.10}$$

III. REDUCED BALLISTIC SYSTEM

At this point, the interior ballistics of a CAD firing are completely described during burning by the following equations.

Motion

$$pA = \frac{w_1}{g_c} \left(\frac{d^2 x}{dt^2} + g \sin\theta \right) \tag{3.1}$$

State

$$p \left[\frac{12\ Ax + U_i}{C} - \left(\eta - \frac{1}{\rho}\right) \right] = 12\ NRT \tag{3.2}$$

Energy Balance

$$C_v C(T_v - T) = \left(\frac{1 + K_1 + K_2}{1 + K_1} \right) \frac{w_1}{2g_c}\ v^2 + \frac{w_1 gx \sin\theta}{g_c} \tag{3.3}$$

Burning Rate

$$r = \frac{dL}{dt} = Bp^n \tag{3.4}$$

Form Function

$$C = \Lambda_1 L + \Lambda_2 L^2 + \Lambda_3 L^3 \tag{3.5}$$

The system is said to have a solution when all the variables are determined as functions of a single parameter such as time, t, piston travel, x, or piston velocity, v. Unfortunately, the system as it stands has no solution in closed form. However, with the aid of the following simplifications, the equations may be reduced to a system for which an analytic solution may be found.

1. The nonlinear burning rate equation (3.4) is replaced with the linear form

$$r = \alpha\ Bp \tag{3.6}$$

where α is chosen so as to give the "best" linear approximation to equation (3.4). One way of achieving this end is to choose α such that the integral

$$\sigma_1 = \int_0^{p_{max}} B^2\ (\alpha p - p^n)^2\ dp$$

is a minimum. This selection of α ensures a "best" fit in the least squares sense to the burning rate over the CAD pressure range.

Carrying out the indicated integration of the above equation, setting $\frac{d\sigma_1}{d\alpha} = o$, and solving for α yields

$$\alpha = \frac{3}{n+2} p_{max}^{n-1} \tag{3.7}$$

where p_{max} is the maximum pressure.

Another way of achieving a "best" linear fit is to choose α such that the integral

$$\sigma_2 = \int_0^{p_{max}} B \mid \alpha p - p^n \mid dp$$

is a minimum. Integrating, setting $\frac{d\sigma_2}{d\alpha} = o$ and solving for α yields

$$\alpha = \left(\frac{p_{max}}{\sqrt{2}}\right)^{n-1} . \tag{3.8}$$

Still another way of obtaining a "best" fit is to choose α such that

$$\int_o^{p_{max}} Bp^n \, dp = \int_o^{p_{max}} \alpha Bp dp$$

in other words, to make the areas under the two burning rate curves equal. Carrying out the integrations and solving for α yields

$$\alpha = \frac{2}{n+1} p_{max}^{n-1} . \tag{3.9}$$

Other "best" fits may be derived in a similar manner. The final decision as to which definition of α should be used is rather arbitrary. Of the several possible definitions, equation (3.9) seems to lead to the best agreement between experimental and calculated values of maximum pressure and ejection velocity. Consequently, it will be taken as the definition of α.

Nomographs depicting the relationship between α, p_{max}, and n expressed in equation (3.9) are given in Figures 1a and 1b in Appendix F. A straight line connecting the given values of any two of the parameters will pass through the corresponding value of the third.

2. θ is assumed to be 0 in equations (3.1) and (3.3). A suitable correction factor is developed in Section VI to compensate for errors introduced by the assumption.

3. The covolume factor, η, is assumed to equal the specific volume of the propellant. Thus, $\eta - \frac{1}{\rho} = 0$ in equation (3.2). Generally, for those CAD's operating under pressures below 10,000 psi the assumption leads to insignificant errors. As a matter of interest, however, a correction factor is derived in Section VI and may be employed if desired.

4. The cubic form function, equation (3.5) is replaced, for the moment, by the linear relationship

$$C = \lambda L . \tag{3.10}$$

(Solutions will be obtained in Section X assuming a quadratic form function.) This is equivalent to replacing the true burning surface at any time by the average burning surface, $\bar{S}$, during the burning regime. Thus,

$$\lambda \equiv \rho \bar{S} \equiv C_b / L_b . \tag{3.11}$$

For constant surface grains the relationship, equation (3.10), is of course exact. Surprisingly, it often leads to quite accurate results in the case of progressive and regressive grains as well.

It should be noted that conditions 2., 3., and 4. need not be included in the list of simplifications; an analytic solution may be obtained merely by employing condition 1. The error, introduced by the conditions, however, is more than offset by the simplicity of the resulting solution. Any useful ballistic system must of necessity be a compromise system in which some of the accuracy is sacrificed in order to permit more rapid solution of problems. The system developed here is no exception to the rule.

Imposing the foregoing conditions, and for conciseness writing $\frac{w_1}{g_c} = W$, the reduced system of equations to be solved is:

During Burning

$$pA = W \frac{d^2x}{dt^2} = W v \frac{dv}{dx} \tag{3.12}$$

$$p \left[\frac{12\ Ax + U_i}{C} \right] = 12\ NRT \tag{3.13}$$

$$C_v C (T_v - T) = \left(\frac{1 + K_1 + K_2}{1 + K_1} \right) \frac{W}{2} v^2 \tag{3.14}$$

$$r = \frac{dL}{dt} = \alpha\ Bp \tag{3.15}$$

$$C = \lambda L \tag{3.16}$$

After Burnt

$$pA = W \frac{d^2x}{dt^2} = W v \frac{dv}{dx} \tag{3.17}$$

$$p \left[\frac{12\ Ax + U_i}{C_b} \right] = 12\ NRT \tag{3.18}$$

$$C_v C_b (T_v - T) = \left(\frac{1 + K_1 + K_2}{1 + K_1} \right) \frac{W}{2} v^2 \tag{3.19}$$

IV. DIMENSIONLESS SOLUTION: $C = \lambda L$

A. During Burning

Combining equations (3.12) and (3.15), recalling that $r = \frac{dL}{dt}$, and $v = \frac{dx}{dt}$, we obtain through integration

$$L = \frac{\alpha BW}{A} v. \tag{4.1}$$

Thus, the distance burned through the grain at any time is proportional to the velocity of the mass in motion. It is here assumed that the mass begins accelerating immediately upon ignition of the propellant, i. e., the value of

p at $t = o$ is zero. In view of the fact that CAD's are essentially smoothbore guns, the assumption seems to be a valid one, at least for horizontal firings. Furthermore, the assumption of a non-zero starting pressure overly complicates the solution; it has therefore not been included in the analysis.

Equation (4.1) permits the reduction of the system (3.12) through (3.16) to the single first order linear differential equation

$$(12\,Ax + U_i)\,\frac{dv}{dx} = 12\,\alpha\,BF\lambda - 6\left(\frac{1 + K_1 + K_2}{1 + K_1}\right)(\gamma - 1)\,A\,v \tag{4.2}$$

where $\gamma \equiv 1 + \frac{NR}{C_v}$ is the "ratio of specific heats", and $F \equiv NRT_v$ is the "impetus" of the propellant.

Considerable simplification results if we define an adjusted ratio of specific heats,

$$\bar{\gamma} \equiv 1 + \left(\frac{1 + K_1 + K_2}{1 + K_1}\right)(\gamma - 1) \tag{4.3}$$

a dimensionless distance variable,

$$X \equiv \frac{12A}{U_i}\,x \tag{4.4}$$

and a dimensionless velocity variable

$$V \equiv \frac{A}{\alpha\,BF\,\lambda}\,v \tag{4.5}$$

for then equation (4.2) may be written

$$(1 + X)\,\frac{dV}{dX} = 1 - \frac{\bar{\gamma} - 1}{2}\,V\,. \tag{4.6}$$

Separating variables in the equation and integrating, remembering that initially $X = V = 0$, X is obtained as a function of V

$$X = \left(\frac{1}{1 - \frac{\bar{\gamma} - 1}{2}\,V}\right)^{\frac{2}{\bar{\gamma} - 1}} - 1\,. \tag{4.7}$$

The equation relates the piston travel to its velocity at any time during burning. Figures 2a and 2b contain graphs of X versus V for various values of $\bar{\gamma}$. (The reason for including values of $\bar{\gamma} \leq 1$ will be explained in Section X.)

To determine the pressure as a function of the piston velocity, let a dimensionless pressure variable, P, be defined as

$$P \equiv \frac{U_i}{12W}\left(\frac{A}{\alpha\, BF\, \lambda}\right)^2 p \ . \tag{4.8}$$

Substituting equations (4.6), (4.7), and (4.8) into equation (3.12) then yields

$$P = V\frac{dV}{dX} = V\left(1 - \frac{\bar{\gamma}-1}{2}V\right)^{\frac{\bar{\gamma}+1}{\bar{\gamma}-1}} . \tag{4.9}$$

Employing the foregoing equation, P versus V is plotted in Figure 3 for various values of $\bar{\gamma}$.

A glance at Figure 3 will show that P goes through a mathematical maximum at some value $V = V_M$. By differentiating P with respect to V in equation (4.9), setting the derivative equal to zero and solving for V, the value of V_M is found to be

$$V_M = \frac{1}{\bar{\gamma}} \ . \tag{4.10}$$

Hence, the value of P at V_M is

$$P_M = V_M\left(1 - \frac{\bar{\gamma}-1}{2}V_M\right)^{\frac{\bar{\gamma}+1}{\bar{\gamma}-1}} = \frac{1}{\bar{\gamma}}\left(\frac{\bar{\gamma}+1}{2\bar{\gamma}}\right)^{\frac{\bar{\gamma}+1}{\bar{\gamma}-1}} . \tag{4.11}$$

To determine whether P_M is actually attained in a particular CAD firing we must first know the values of two parameters: V_b, the value of V at grain burnout, and V_{m1}, a tentative value of V at completion of the piston stroke.

The value of V_b is found from equations (4.1) and (4.5) to be

$$V_b = \frac{A}{\alpha\, BF\, \lambda}v_b = \left(\frac{A}{\alpha\, B}\right)^2 \frac{L_b}{F\, \lambda\, W} \ . \tag{4.12}$$

The tentative dimensionless ejection velocity is obtained by a rearrangement of equation (4.7) as

$$V_{m1} = \frac{2}{\bar{\gamma} - 1}\left[1 - \left(\frac{1}{1 + \frac{12\,Ax_m}{U_i}}\right)^{\frac{\bar{\gamma}-1}{2}}\right] \qquad (4.13)$$

where x_m is the stroke length. (V_{m1} is a tentative value since equation (4.13) is valid only if burning takes place throughout piston travel.)

The largest value of P, call it P_μ, attained in a particular firing is then

$$P_\mu = V_\mu \left(1 - \frac{\bar{\gamma}-1}{2}\, V_\mu\right)^{\frac{\bar{\gamma}+1}{\bar{\gamma}-1}} \qquad (4.14)$$

where V_μ is the smallest of the three values, V_M, V_{m1}, and V_b. Stated more concisely

$$V_\mu = \min\,(V_M,\ V_{m1},\ V_b)\ . \qquad (4.15)$$

The temperature, T, may also be determined as a function of the velocity. Substituting the values of the dimensionless variables into equation (3.13) with subsequent rearrangement yields

$$T = T_v\left(1 - \frac{\bar{\gamma}-1}{2}\, V\right). \qquad (4.16)$$

B. <u>After Burnt</u>

Combining equations (3.17), (3.18), and (3.19), the differential equation governing the ballistics after grain burnout is found to be

$$(12\,Ax + U_i)\, v\, \frac{dv}{dx} = \frac{12\,FAC_b}{W} - 6A\left(\frac{1 + K_1 + K_2}{1 + K_1}\right)(\gamma - 1)\, v^2\ . \qquad (4.17)$$

Substituting the dimensionless variables defined by equations (4.3), (4.4) and (4.5) the governing equation simplifies to

$$(1 + X)\, V\, \frac{dV}{dX} = V_b - \frac{\bar{\gamma}-1}{2}\, V^2\ . \qquad (4.18)$$

Separating the variables and integrating from conditions at grain burnout, the dimensionless velocity after burnt is obtained in the form

$$V = \frac{2}{\bar{\gamma} - 1} \sqrt{\frac{\bar{\gamma} - 1}{2} V_b \left[1 - \left(1 - \frac{\bar{\gamma} - 1}{2} V_b\right)\left(\frac{1 + X_b}{1 + X}\right)^{\bar{\gamma} - 1}\right]} \tag{4.19}$$

Now, let

$$\left(\frac{1 + X_b}{1 + X_m}\right) \equiv \nu \tag{4.20}$$

and

$$\nu^{\bar{\gamma} - 1} \equiv \phi . \tag{4.21}$$

Then equation (4.19) may be used to determine the dimensionless ejection velocity,

$$V_{m2} = \frac{2}{\bar{\gamma} - 1} \sqrt{\frac{\bar{\gamma} - 1}{2} V_b \left[1 - \left(1 - \frac{\bar{\gamma} - 1}{2} V_b\right) \phi\right]} \tag{4.22}$$

provided $V_{m1} > V_b$. In Figure 4 are plotted values of ϕ versus ν for various values of $\bar{\gamma}$; Figure 5 contains graphs relating the quantities $\frac{\bar{\gamma} - 1}{2} V_{m2}$, $\frac{\bar{\gamma} - 1}{2} V_b$, and ϕ.

The dimensionless pressure, P, and the gas temperature, T, after propellant burnout are found by algebraic manipulation of equations (3.17), (3.18), and (3.19) to be

$$P = \left(\frac{1 - \frac{\bar{\gamma} - 1}{2} V_b}{V_b}\right)^{\frac{1}{\bar{\gamma} - 1}} \left(V_b - \frac{\bar{\gamma} - 1}{2} V^2\right)^{\frac{\bar{\gamma}}{\bar{\gamma} - 1}} \tag{4.23}$$

and

$$T = T_v \left(1 - \frac{\bar{\gamma} - 1}{2} \frac{V^2}{V_b}\right) . \tag{4.24}$$

V. SUMMARY OF SOLUTION EQUATIONS; $C = \lambda L$

Except for the time variable, t, which cannot be found in terms of elementary functions,[1] the solution to the ballistic equations (3.12) through (3.19) is now complete. The solution equations are collected here for ease of reference.

Define the dimensionless quantities

$$\bar{\gamma} \equiv 1 + \left(\frac{1 + K_1 + K_2}{1 + K_1}\right)(\gamma - 1) \tag{5.1}$$

$$X \equiv \frac{12\, A}{U_i}\, x \tag{5.2}$$

$$V \equiv \frac{A}{\alpha\, BF\, \lambda}\, v \tag{5.3}$$

and

$$P \equiv \frac{U_i}{12W}\left(\frac{A}{\alpha\, BF\, \lambda}\right)^2 p\,. \tag{5.4}$$

The solution during burning is

$$V = \frac{2}{\bar{\gamma} - 1}\left[1 - \left(\frac{1}{1 + X}\right)^{\frac{\bar{\gamma} - 1}{2}}\right] \tag{5.5}$$

$$P = V\left(1 - \frac{\bar{\gamma} - 1}{2}\, V\right)^{\frac{\bar{\gamma} + 1}{\bar{\gamma} - 1}} \tag{5.6}$$

$$T = T_v\left(1 - \frac{\bar{\gamma} - 1}{2}\, V\right) \tag{5.7}$$

$$\text{maximum } P = P_\mu = V_\mu\left(1 - \frac{\bar{\gamma} - 1}{2}\, V_\mu\right)^{\frac{\bar{\gamma} + 1}{\bar{\gamma} - 1}} \tag{5.8}$$

[1] Actually the system (3.12)-(3.16) leads to infinite t in the case $P(0) = 0$. See Hirschfelder, Reference (b), page 15, for a discussion of the problem.

where V_μ is the smallest of the three values

$$V_M = \frac{1}{\bar{\gamma}} \tag{5.9}$$

$$V_{m1} = \frac{2}{\bar{\gamma} - 1} \left[1 - \left(\frac{1}{1 + X_m} \right)^{\frac{\bar{\gamma} - 1}{2}} \right] \tag{5.10}$$

and

$$V_b = \left(\frac{A}{\alpha B} \right)^2 \frac{L_b}{F \lambda W} . \tag{5.11}$$

If $V_b \geq V_{m1}$, burning takes place during the entire length of the piston stroke and no additional equations are required. If $V_b < V_{m1}$, burnout occurs before the end of the stroke. The following equations are applicable during the period after burnt.

$$V = \frac{2}{\bar{\gamma} - 1} \sqrt{\frac{\bar{\gamma} - 1}{2} V_b \left[1 - \left(1 - \frac{\bar{\gamma} - 1}{2} V_b \right) \left(\frac{1 + X_b}{1 + X} \right)^{\bar{\gamma} - 1} \right]} \tag{5.12}$$

$$P = \left(\frac{1 - \frac{\bar{\gamma} - 1}{2} V_b}{V_b} \right)^{\frac{1}{\bar{\gamma} - 1}} \left(V_b - \frac{\bar{\gamma} - 1}{2} V^2 \right)^{\frac{\bar{\gamma}}{\bar{\gamma} - 1}} \tag{5.13}$$

and

$$T = T_v \left(1 - \frac{\bar{\gamma} - 1}{2} \frac{V^2}{V_b} \right) . \tag{5.14}$$

VI. CORRECTION FACTORS FOR NONHORIZONTAL FIRING AND COVOLUME EFFECTS

A. Nonhorizontal Firing

The preceding analysis is based on the assumption that $\theta = 0$ in the equations of motion and energy balance. This is of course not always the case. θ may vary over the range $-90^\circ \leq \theta \leq +90^\circ$. The magnitude of the error introduced by the assumption is not large, being rarely more than 3 or 4 per cent in the most extreme cases, but a fairly reliable correction factor may be derived and is done so in this section.

Consider the system

$$pA = W\left(\frac{dv}{dt} + g \sin \theta\right) \tag{6.1}$$

$$p\left[\frac{12\, Ax + U_i}{C}\right] = 12\, NRT \tag{6.2}$$

$$C_v C(T_v - T) = \left(\frac{1 + K_1 + K_2}{1 + K_1}\right) \frac{W}{2}\, v^2 + W\, gx \sin \theta \tag{6.3}$$

$$\frac{dL}{dt} = \alpha B p \tag{6.4}$$

$$C = \lambda L\ . \tag{6.5}$$

As in the case of the friction force, F_f, it is assumed that, in the large, the ballistic effects of the term, $g \sin \theta$, may be simulated by replacing the term with one that is proportional to the pressure, p. The assumption permits us to write

$$g \sin \theta = \frac{K_3 A p}{(1 + K_3) W} \tag{6.6}$$

where K_3 is a small constant to be evaluated later.

Substituting equation (6.6) into equation (6.1) and rearranging yields

$$pA = (1 + K_3)\, W\, v\, \frac{dv}{dx}\ . \tag{6.7}$$

Also, the term $gx \sin \theta$ may be written with the aid of equation (6.6)

$$gx \sin \theta = \int_0^x g \sin \theta\, dx = \int_0^x \frac{K_3\, pA dx}{(1 + K_3) W}\ . \tag{6.8}$$

Substituting equation (6.7) into equation (6.8) and integrating we obtain

$$gx \sin \theta = \frac{K_3}{2}\ v^2 \tag{6.9}$$

which when combined with equation (6.3) produces

$$C_v C(T_v - T) = \frac{\dfrac{1 + K_1 + K_2}{1 + K_1} + K_3}{1 + K_3} (1 + K_3) \frac{W}{2} v^2 \quad .$$

However, since K_3 is small, the foregoing equation may be very closely approximated by

$$C_v C(T_v - T) = \left(\frac{1 + K_1 + K_2}{1 + K_1}\right) (1 + K_3) \frac{W}{2} v^2 \; . \tag{6.10}$$

A comparison of equations (6.7) and (6.10) indicates that the net result of the assumption is to reduce the system (6.1)-(6.5) to the system (3.12)-(3.16) with $(1 + K_3)W$ replacing W. Experience indicates that if $\frac{W}{A} g \sin \theta$ is less than 5 or 6 per cent of p_{max} the assumption is a valid one.

We seek now the ballistic effects of a small change in the mass W. Consider two CAD firings identical in all respects except that an effective mass W_1 is accelerated in the first, and a mass $W_2 = (1 + K_3)W_1$ is accelerated in the second. Assuming that burning takes place during the greater part of the stroke, it follows from equations (5.2), (5.5), and (5.6) that at any given value of the piston displacement, x, the quantities P and V are the same for both firings. Thus, it is concluded from equation (5.4) that at the given value of x

$$\frac{p_1}{W_1 \alpha_1^{\,2}} = \frac{p_2}{W_2 \alpha_2^{\,2}} \tag{6.11}$$

where subscripts 1 and 2 refer to firing numbers one and two, respectively. Since by definition $\alpha = \frac{2}{n + 1} p_{max}^{\,n-1}$, the above may be arranged to yield

$$p_2 = p_1 (1 + K_3) \left(\frac{\alpha_2}{\alpha_1}\right)^2 = p_1 (1 + K_3) \left(\frac{p_{max\,1}}{p_{max\,2}}\right)^{2-2n} . \tag{6.12}$$

The equation must hold for $p_1 = p_{max\ 1}$ and $p_2 = p_{max\ 2}$; hence making the substitutions and solving for $p_{max\ 2}$ we have

$$p_{max\ 2} = (1 + K_3)^{\frac{1}{3 - 2n}} p_{max\ 1} \tag{6.13}$$

or since $K_3 << 1$, using two terms of the binomial expansion gives

$$p_{max\ 2} \approx \left(1 + \frac{K_3}{3 - 2n}\right) p_{max\ 1} . \tag{6.14}$$

The effect then of multiplying the mass W by a factor $(1 + K_3)$ is to multiply the maximum pressure approximately by a factor $\left(1 + \frac{K_3}{3 - 2n}\right)$.

To determine velocity effects we note that equation (5.3) permits us to write

$$\frac{v_1}{\alpha_1} = \frac{v_2}{\alpha_2} \tag{6.15}$$

at any given value of x. Using the definition of α, the above then yields

$$v_2 = \frac{\alpha_2}{\alpha_1} v_1 = \left(\frac{p_{max 2}}{p_{max 1}}\right)^{n-1} v_1 = (1 + K_3)^{\frac{n-1}{3-2n}} v_1 . \tag{6.16}$$

The relation must hold at $x = x_m$, hence again using two terms of the binomial expansion we have

$$v_{m2} \approx \left(1 - \frac{1 - n}{3 - 2n} K_3\right) v_{m1} . \tag{6.17}$$

(As a rather interesting by-product of the analysis it is noted that:

1. For a burning rate exponent of one, the ejection velocity is independent of changes in the accelerated mass;

2. For exponents greater than one, the velocity increases with increasing W;

3. Exponents much larger than one should be avoided because of the extreme pressure sensitivity as $n \to 1.5$.)

The value of K_3 to be used to compensate for nonhorizontal firing is obtained from equation (6.9) by setting $v = v_{m2}$ and $x = x_m$. Thus

$$K_3 = \frac{2\, g x_m \sin\theta}{v_{m2}^2} \approx \frac{2\, g x_m \sin\theta}{v_{m1}^2} \quad . \tag{6.18}$$

B. Covolume Effects

In the large, the ballistics effects of the covolume term may be simulated in the following manner. Consider the equation of state (3.2) in the form

$$p \left(\frac{12\, Ax + U_i}{C}\right) - p\,(\eta - \frac{1}{\rho}) = 12\, NRT \; . \tag{6.19}$$

If now the term $p(\eta - \frac{1}{\rho})$ is replaced by an average value $\frac{p_{max}}{2}(\eta - \frac{1}{\rho})$, instead of assuming $\eta - \frac{1}{\rho} = 0$, equations (3.12)-(3.16) reduce to the single differential equation

$$(12\, Ax + U_i)\frac{dv}{dx} = 12\,\alpha BF'\lambda - 6\left(\frac{1 + K_1 + K_2}{1 + K_1}\right)(\gamma - 1)\, Av \; , \tag{6.20}$$

where

$$F' = \left[1 + \frac{p_{max}}{24F}(\eta - \frac{1}{\rho})\right] F \; . \tag{6.21}$$

Equation (6.20) is identical to equation (4.2) with the exception that F' replaces F. Furthermore, the term $\frac{p_{max}}{24F}(\eta - \frac{1}{\rho})$ is generally less than 0.02 for most CAD's. Thus, the error introduced by neglecting the covolume term may be approximately compensated for by a small increase in the impetus, F, of the propellant.

To determine the ballistic effects of a small increase in F, again consider two CAD firings having identical ballistic parameters except that the impetus of the first is F_1 and that of the second is $F_2 = (1 + K_4)F_1$, where $K_4 << 1$. As in the preceding analysis, it is concluded that at any given value of the displacement, x,

$$\frac{p_1}{F_1^{\,2}\alpha_1^{\,2}} = \frac{p_2}{F_2^{\,2}\alpha_2^{\,2}} \tag{6.22}$$

where again subscripts 1 and 2 refer to firing numbers one and two, respectively. Employing the definition $\alpha = \frac{2}{n+1}\, p_{max}^{\,n-1}$, the relationship between the two maximum pressures is found to be

$$p_{max\,2} = (1 + K_4)^{\frac{2}{3-2n}}\, p_{max\,1} \tag{6.23}$$

or, approximately,

$$p_{max\,2} = \left(1 + \frac{2K_4}{3-2n}\right) p_{max\,1} \quad . \tag{6.24}$$

To determine the effect on the ejection velocity it follows from equation (5.3) that

$$\frac{v_{m1}}{\alpha_1 F_1} = \frac{v_{m2}}{\alpha_2 F_2} \tag{6.25}$$

so that using equations (6.22) and (6.23)

$$v_{m2} = \sqrt{\frac{p_{max\,2}}{p_{max\,1}}}\, v_{m1} = (1 + K_4)^{\frac{1}{3-2n}}\, v_{m1} \quad . \tag{6.26}$$

Hence,

$$v_{m2} \approx \left(1 + \frac{K_4}{3-2n}\right) v_{m1} \quad . \tag{6.27}$$

The value of K_4 to be used to compensate for covolume effects is determined from equation (6.21).

$$K_4 = \frac{p_{max\,2}}{24F}\left(\eta - \frac{1}{\rho}\right) \approx \frac{p_{max\,1}}{24F}\left(\eta - \frac{1}{\rho}\right) \quad . \tag{6.28}$$

C. Combined Correction Factor

The corrections for nonhorizontal firing and neglected covolume may be combined into a single correction factor. The following procedure is suggested.

First assume $\theta = 0$ and $\eta = \frac{1}{\rho}$. Calculate the maximum pressure, p_{max}, and ejection velocity, v_m, using the method described in the following section. The corrected maximum pressure, p^*_{max}, and the corrected ejection velocity, v^*_m, are determined from

$$p^*_{max} = \left(1 + \frac{1}{3 - 2n} K_3\right)\left(1 + \frac{2}{3 - 2n} K_4\right) p_{max} \qquad (6.29)$$

and

$$v^*_m = \left(1 - \frac{1 - n}{3 - 2n} K_3\right)\left(1 + \frac{1}{3 - 2n} K_4\right) v_m \qquad (6.30)$$

where

$$K_3 = \frac{2\, g x_m \sin\theta}{v_m^2}$$

and

$$K_4 = \frac{p_{max}\left(\eta - \frac{1}{\rho}\right)}{24F} .$$

VII. GRAPHICAL DETERMINATION OF MAXIMUM PRESSURE AND EJECTION VELOCITY; $C = \lambda L$

The fact that the parameter, α, is itself a function of the maximum pressure precludes an explicit determination of the maximum pressure and ejection velocity. However, a rapidly converging trial-and-error procedure may be devised utilizing the equations summarized in Section V. The procedure requires more than one iteration only if the maximum pressure occurs at "burnt".

We begin by defining the parameters:

$$q_1 \equiv \frac{U_i}{12W}\left(\frac{A}{BF\lambda}\right)^2 \qquad (7.1)$$

$$q_2 \equiv \frac{A}{BF\lambda} \qquad (7.2)$$

$$q_3 \equiv \left(\frac{A}{B}\right)^2 \frac{L_b}{F\lambda W} \qquad (7.3)$$

$$Q_1 = q_1/\alpha^2 \tag{7.4}$$

and

$$Q_2 = q_2/\alpha \quad . \tag{7.5}$$

The dimensionless variables defined by equations (5.3), (5.4), and (5.11) are then written simply

$$P = Q_1 p = q_1 p/\alpha^2 \tag{7.6}$$

$$V = Q_2 v = q_2 v/\alpha \tag{7.7}$$

and

$$V_b = q_3/\alpha^2 \quad . \tag{7.8}$$

Now, p_{max} may be eliminated from the two equations

$$P_\mu = q_1 \, p_{max}/\alpha^2$$

and

$$\alpha = \frac{2}{n+1} \, p_{max}^{n-1}$$

to yield α as a function of the burning rate exponent, n, and the ratio P_μ/q_1. Thus

$$\alpha = \left[\left(\frac{2}{n+1}\right)\left(\frac{P_\mu}{q_1}\right)^{n-1} \right]^{\frac{1}{3-2n}} \quad . \tag{7.9}$$

The relationship between α, n, and P_μ/q_1 expressed in the foregoing equation is shown nomographically in Figures 6a and 6b. (Again, a straight line connecting the values of any two of the parameters passes through the corresponding value of the third.)

The trial-and-error procedure may now be outlined.

Step 1

a. Using the given data for a particular firing, calculate the values of the parameters

$$\lambda = C_b / L_b$$

$$\bar{\gamma} = \left(\frac{1 + K_1 + K_2}{1 + K_1}\right) (\gamma - 1) + 1$$

$$U_i = U_c - C_b / \rho$$

$$W = (1 + K_1)\, w / g_c$$

$$q_1 = \frac{U_i}{12W} \left(\frac{A}{BF\lambda}\right)^2$$

$$q_2 = \frac{A}{BF\lambda}$$

$$q_3 = \left(\frac{A}{B}\right)^2 \frac{L_b}{F\lambda W}$$

$$V_M = \frac{1}{\bar{\gamma}}$$

and

$$X_m = 12\, Ax_m / U_i \quad .$$

b. Obtain $V_{m1} = f(X_m, \bar{\gamma})$ from Figure 2.

Step 2

a. Assume a value of P_μ. (Initially, this is done by selecting the value of P from Figure 3 corresponding to the smaller of the two values V_{m1} and V_M as determined in Step 1.)

b. Calculate P_μ/q_1.

c. Determine α from Figure 6 corresponding to the burning-rate exponent, n, and the ratio, P_μ/q_1.

d. Calculate $V_b = q_3/\alpha^2$.

e. Determine $V_\mu = \min\,(V_\mu, V_b, V_{m1})$. Select the value of $P_\mu = f(V_\mu, \bar{\gamma})$ from Figure 3.

f. If the assumed value of P_μ equals the calculated value, go on to Step 3; if not repeat Step 2 using the calculated value of P_μ.

Step 3

a. Calculate

$$Q_1 = q_1/\alpha^2$$

and

$$Q_2 = q_2/\alpha .$$

b. Calculate the maximum pressure using $p_{max} = P_\mu/Q_1$.

Step 4

Compare the values of V_b and V_{m1}.

Case A: $V_b \geq V_{m1}$. Calculate the ejection velocity using $v_m = V_{m1}/Q_2$.

Case B: $V_b < V_{m1}$.

a. Determine $X_b = f(V_b, \bar{\gamma})$ from Figure 2.

b. Calculate $\nu = \dfrac{1 + X_b}{1 + X_m}$.

c. Determine $\phi = f(\nu, \bar{\gamma})$ from Figure 4.

d. Determine $\dfrac{\bar{\gamma} - 1}{2} V_{m2} = f\left(\dfrac{\bar{\gamma} - 1}{2} V_b, \phi\right)$ from Figure 5.

e. Calculate $V_{m2} = \frac{2}{\bar{\gamma} - 1}\left(\frac{\bar{\gamma} - 1}{2} V_{m2}\right)$.

f. Calculate the ejection velocity using $v_m = V_{m2}/Q_2$.

<u>Step 5</u>

The correction factors derived in Section VI may be applied if desired.

Calculate

$$K_3 = \frac{2\, g x_m \sin\theta}{v_m^2}$$

and

$$K_4 = \frac{p_{\max}\left(\eta - \frac{1}{\rho}\right)}{24F} .$$

The corrected values of maximum pressure and ejection velocity are then determined from

$$p^*_{\max} = \left(1 + \frac{1}{3 - 2n} K_3\right)\left(1 + \frac{2}{3 - 2n} K_4\right) p_{\max}$$

and

$$v^*_m = \left(1 - \frac{1 - n}{3 - 2n} K_3\right)\left(1 + \frac{1}{3 - 2n} K_4\right) v_m .$$

Sample Problem 1.

Given the following data

$A = 4.92 \text{ in}^2$

$B = 0.02101 \left(\frac{\text{in}^2}{\text{lb}_f}\right)^n$ in/sec

$C_b = 0.161\ \text{lb}_m$[1]

[1] C_b includes the main charge weight plus the "equivalent igniter weight." The latter, denoted by C_i, is defined as $C_i = (F_i/F)I$, where F_i is the impetus of the igniter charge and I is the igniter charge weight.

$$F = 285{,}500 \ \mathrm{ft \cdot lb_f / lb_m}$$

$$K_1 = 0.10$$

$$K_2 = 0.25$$

$$L_b = 0.075 \ \mathrm{in}$$

$$n = 0.47$$

$$U_c = 159.1 \ \mathrm{in^3}$$

$$w = 360 \ \mathrm{lb_m}$$

$$x_m = 2.725 \ \mathrm{ft}$$

$$\theta = 70.5 \ \mathrm{degrees}$$

$$\gamma = 1.235$$

$$\rho = 0.06 \ \mathrm{lb_m / in^3}$$

$$\eta = 30 \ \mathrm{in^3 / lb_m}$$

determine the maximum pressure and ejection velocity.

The given data are listed in Table 1. The required parameters are then calculated and the values $V_M = 0.7764$ and $V_{m1} = 0.6800$ are obtained. The initial assumed value of P_μ is taken to be $P(V_{m1}) = 0.295$. The assumed and calculated values of P_μ are found to be equal after one iteration and the maximum pressure, p_{max}, is calculated to be 1496 psi. Since $V_b > V_{m1}$, the ejection velocity is $V_{m1}/Q_2 = 49.8$ ft/sec. The correction factors of Section VI are then applied leading to $p^*_{max} = 1550$ psi and $v^*_m = 49.0$ ft/sec. The values compare favorably to those obtained through numerical integration of Equations (3.12)-(3.16) namely, 1607 psi and 51.9 ft/sec.

An estimate of the errors inherent in the graphical method can be made. Assuming P_μ, V_{m1}, and α are determinable from the graphs to within ± 2 per cent of their correct values, the maximum errors in p^*_{max} and v^*_m are about

Table 1

DETERMINATION OF MAXIMUM PRESSURE AND EJECTION VELOCITY; $C = \lambda L$

Sample Problem 1.

Given Values	
A	$= 4.92\ in^2$
B	$= 0.02101\ (in^2/lb_f)^n$ in/sec
C_b	$= 0.161\ lb_m$
F	$= 2.855^5\ ft\cdot lb_f/lb_m$
K_1	$= 0.1$
K_2	$= 0.25$
L_b	$= 0.075$ in
n	$= 0.47$
U_c	$= 159.1\ in^3$
w	$= 360\ lb_m$
x_m	$= 2.725$ ft
γ	$= 1.235$
θ	$= 70.5°$
$\sin\theta$	$= 0.94264$
ρ	$= 0.06\ lb_m/in^3$
η	$= 30\ in^3/lb_m$

Step 1

$\lambda = C_b/L_b = 2.1467$

$\bar{\gamma} = \left(\frac{1+K_1+K_2}{1+K_1}\right)(\gamma - 1) + 1 = 1.288$

$U_i = U_c - C_b/\rho = 156.42$

$W = (1+K_1)w/g_c = 12.308$

$q_1 = \frac{U_i}{12W}\left(\frac{A}{BF\lambda}\right)^2 = 1.5462^{-7}$

$q_2 = \frac{A}{BF\lambda} = 3.8209^{-4}$

$q_3 = \left(\frac{A}{B}\right)^2 \frac{L_b}{F\lambda W} = 5.4522^{-4}$

$V_N = 1/\bar{\gamma} = 0.7764$

$X_m = 12\ Ax_m/U_i = 1.029$

$V_{m1} = f(X_m, \bar{\gamma})$; Figure 2 = 0.68

Step 2	Trial Number 1	2	3	
P_μ =	0.295			Assumed; Figure 3
P_μ/q_1 =	1.908^{6}			
α =	0.028			Figure 6 $n = 0.47$
α^2 =	7.84^{-4}			
V_b =	0.6954			$V_b = q_3/\alpha^2$
V_N =	0.7764			From Step 1
V_{m1} =	0.6800			From Step 1
V_μ =	0.6800			$V_\mu = \min(V_b, V_N, V_{m1})$
P_μ =	0.295			$P_\mu = f(V_\mu, \bar{\gamma})$; Figure 3

Step 3	
$Q_1 = 1.972^{-4}$	$Q_1 = q_1/\alpha^2$
$Q_2 = 1.365^{-2}$	$Q_2 = q_2/\alpha$
$\phi_{max} = 1496$	$\phi_{max} = P_\mu/Q_1$

Step 4

Case A: $V_b \geq V_{m1}$? $\boxed{\times}$ $v_m = V_{m1}/Q_2 = 49.8$

Case B: $V_b < V_{m1}$? $\square$

$X_b = f(V_b, \bar{\gamma})$; Figure 2 =

$\nu = (1+X_b)/(1+X_m)$ =

$\phi = f(\nu, \bar{\gamma})$; Figure 4 =

$\frac{\bar{\gamma}-1}{2} V_{m2} = f\left(\frac{\bar{\gamma}-1}{2} V_b, \phi\right)$; Figure 5 =

$V_{m2} = \frac{2}{\bar{\gamma}-1}\left(\frac{\bar{\gamma}-1}{2}\right)V_{m2}$ =

$v_m = V_{m2}/Q_2$ =

Step 5

$$K_3 = \frac{2\,g x_m \sin\theta}{v_m^2} = 0.067$$

$$K_4 = \frac{\phi_{max}\left(\eta - \frac{1}{\rho}\right)}{24F} = 0.003$$

$$\phi^*_{max} = \left(1 + \frac{1}{3-2n}K_3\right)\left(1 + \frac{2}{3-2n}K_4\right)\phi_{max} = 1550 \text{ psi}$$

$$v^*_m = \left(1 - \frac{1-n}{3-2n}K_3\right)\left(1 + \frac{1}{3-2n}K_4\right)v_m = 49.0 \text{ ft/sec}$$

(Note: a^b denotes $a \times 10^b$)

± 6 per cent and ± 4 per cent respectively. This observation follows from the act that $p_{max} = \frac{P_\mu}{q_1}\alpha^2$ and $v_m = \frac{V_{m1}}{q_2}\alpha$.

Sample Problem 2.

Given the following data

$A = 0.7854\ in^2$

$B = 0.01741\ (in^2/lb_f)^n\ in/sec$

$C_b = 0.009304\ lb_m$

$F = 362{,}300\ ft \cdot lb_f/lb_m$

$K_1 = 0.1$

$K_2 = 0.25$

$L_b = 0.055\ in$

$n = 0.51$

$U_c = 6\ in^3$

$w = 2100\ lb_m$

$x_m = 1\ ft$

$\gamma = 1.215$

$\theta = 0°$

$\rho = 0.06146\ lb_m/in^3$

$\eta = 30\ in^3/lb_m$

determine the maximum pressure and ejection velocity.

The given data are listed in Table 2. In this case, the maximum pressure occurs at burnt so that five iterations are required to obtain convergence of the P_μ values. The results, $p^*_{max} = 6357$ psi and $v^*_m = 8.7$ ft/sec again compare favorably with the values 6619 psi and 8.9 ft/sec obtained from numerical integration.

VIII. DETERMINATION OF CHARGE WEIGHT AND WEB SIZE TO MEET GIVEN PERFORMANCE SPECIFICATIONS

A. Given p_{max} and v_m

If it is specified that:

a. The propellant shall burn throughout the piston stroke, i. e., $V_b \geq V_{m1}$.

and

b. p_{max} shall be a mathematical maximum, i. e., $V_{m1} \geq V_M = \frac{1}{\gamma}$, the charge weight and web size required to produce a given maximum pressure and ejection velocity may usually be calculated explicitly. Conditions a. and b. are not, in general, serious restrictions. They are usually called for in most CAD applications.

Let U_c be the volume of the empty chamber. By definition of initial free volume we then have, using equation (5.2)

$$U_i = U_c - C_b/\rho = \frac{12\ Ax_m}{X_m} \tag{8.1}$$

and solving for C_b

$$C_b = \rho\left(U_c - \frac{12\ Ax_m}{X_m}\right) . \tag{8.2}$$

Condition a. above permits equation (5.3) to be rearranged to give

$$\lambda = \frac{C_b}{L_b} = \left(\frac{A}{\alpha\ BF}\right)\frac{v_m}{V_{m1}} \tag{8.3}$$

Table 2

DETERMINATION OF MAXIMUM PRESSURE AND EJECTION VELOCITY: $C = \lambda L$

Sample Problem 2.

Given Values

A	$= 0.7854$ in^2
B	$= 0.01741$ (in^2/lb$_f$)^n in/sec
C_b	$= 0.009304$ lb$_m$
F	$= 3.623^5$ ft·lb$_f$/lb$_m$
K_1	$= 0.10$
K_2	$= 0.25$
L_b	$= 0.055$ in
n	$= 0.51$
U_c	$= 6$ in^3
w	$= 2100$ lb$_m$
x_m	$= 1$ ft
γ	$= 1.215$
θ	$= 0$ degrees
$\sin\theta = 0$	
ρ	$= 0.06146$ lb$_m$/in^3
η	$= 30$ in^3/lb$_m$

Step 1

$\lambda = C_b / L_b = 0.1691$

$\bar{\gamma} = \left(\frac{1 + K_1 + K_2}{1 + K_1}\right)(\gamma - 1) + 1 = 1.2639$

$U_i = U_c - C_b/\rho = 5.848$

$W = (1 + K_1) w / g_c = 71.797$

$q_1 = \frac{U_i}{12W}\left(\frac{A}{BF\lambda}\right)^2 = 3.680^{-9}$

$q_2 = \frac{A}{BF\lambda} = 7.364^{-4}$

$q_3 = \left(\frac{A}{B}\right)^2 \frac{L_b}{F\lambda W} = 2.545^{5}$

$V_N = 1/\bar{\gamma} = 0.7911$

$X_m = 12\, Ax_m / U_i = 1.612$

$V_{m1} = f(X_m, \bar{\gamma})$; Figure 2 $= 0.91$

Trial Numbers

Step 2	1	2	3	4	5	
P_μ =	0.3	0.126	0.0882	0.0753	0.0715	Assumed; Figure 3
P_μ / q_1 =	8.152^7	3.424^7	2.391^7	2.046^7	1.943^7	
α =	1.3^{-2}	1.6^{-2}	1.75^{-2}	1.8^{-2}	1.8^{-2}	Figure 6; $n = 0.51$
α^2 =	1.69^{-4}	2.560^{-4}	3.063^{-4}	3.240^{-4}	3.240^{-4}	
V_b =	0.1506	0.0994	0.0831	0.0785	0.0785	$V_b = q_3/\alpha^2$
V_N =	0.7911	0.7911	0.7911	0.7911	0.7911	From Step 1
V_{m1} =	0.91	0.91	0.91	0.91	0.91	From Step 1
V_μ =	0.1506	0.0994	0.0831	0.0785	0.0785	$V_\mu = \min(V_b, V_N, V_{m1})$
P_μ =	0.126	0.0882	0.0753	0.0715	0.0715	$P_\mu = f(V_\mu, \bar{\gamma})$; Figure 3

Step 3

Q_1	$= 1.136^{-5}$	Q_1	$= q_1/\alpha^2$
Q_2	$= 4.091^{-2}$	Q_2	$= q_2/\alpha$
p_{max}	$= 6294$	p_{max}	$= P_\mu / Q_1$

Step 4

Case A: $V_b \geq V_{m1}$? ☐ $\quad v_m = V_{m1}/Q_2$

Case B: $V_b < V_{m1}$? ☒

$X_b = f(V_b, \bar{\gamma})$; Figure 2 $= 0.0785$

$\nu = (1 + X_b)/(1 + X_m) = 0.4129$

$\phi = f(\nu, \bar{\gamma})$; Figure 4 $= 0.79$

$\frac{\bar{\gamma}-1}{2} V_{m2} = f\left(\frac{\bar{\gamma}-1}{2} V_b, \phi\right)$; Figure 5 $= 0.047$

$V_{m2} = \frac{2}{\bar{\gamma}-1}\left(\frac{\bar{\gamma}-1}{2} V_{m2}\right) = 0.3561$

$v_m = V_{m2}/Q_2 = 8.7$

Step 5

$$K_3 = \frac{2 F x_m \sin\theta}{v_m^2} = 0$$

$$K_4 = \frac{p_{max}\left(\eta - \frac{1}{\rho}\right)}{24 F} = 0.0099$$

$$p^*_{max} = \left(1 + \frac{1}{3-2n} K_3\right)\left(1 + \frac{2}{3-2n} K_4\right) p_{max} = 6357 \text{ psi}$$

$$v^*_m = \left(1 - \frac{1-n}{3-2n} K_3\right)\left(1 + \frac{1}{3-2n} K_4\right) v_m = 8.7 \text{ ft/sec}$$

(Note: a^b denotes $a \times 10^b$)

or

$$L_b = \left(\frac{\alpha BF}{A}\right)\frac{V_{m1}}{v_m} C_b \quad . \tag{8.4}$$

Now, X_m is related to V_{m1} by the expression

$$X_m = \left(\frac{1}{1 - \frac{\bar{\gamma} - 1}{2} V_{m1}}\right)^{\frac{2}{\bar{\gamma} - 1}} - 1 \tag{8.5}$$

hence, if V_{m1} can be determined from the given conditions, equations (8.2) and (8.4) determine the required charge weight and web size. We proceed, then, to determine V_{m1} as a function of the maximum pressure and ejection velocity.

Combining equations (7.1), (7.2), (7.4), and (7.5) we obtain the relation

$$Q_1 = \frac{U_i}{12\, W} Q_2^{\,2} \quad . \tag{8.6}$$

Multiplying numerator and denominator of the right-hand member by v_m^2 and using equation (7.7), this may be written

$$Q_1 = \frac{U_i\, V_{m1}^{\,2}}{12\, W v_m^{\,2}} \quad . \tag{8.7}$$

By condition b. above, equations (4.11) and (7.6) may be combined yielding

$$Q_1\, p_{\max} = \frac{1}{\bar{\gamma}}\left(\frac{\bar{\gamma} + 1}{2\,\bar{\gamma}}\right)^{\frac{\bar{\gamma} + 1}{\bar{\gamma} - 1}} \tag{8.8}$$

Combining equations (8.7) and (8.8) and substituting for U_i its equivalent, $12\, Ax_m / X_m$, we obtain

$$\frac{p_{\max}\, Ax_m V_{m1}^2}{W\, v_m^2\, X_m} = \frac{1}{\bar{\gamma}}\left(\frac{\bar{\gamma} + 1}{2\,\bar{\gamma}}\right)^{\frac{\bar{\gamma} + 1}{\bar{\gamma} - 1}} \tag{8.9}$$

which, when solved for X_m yields

$$X_m = \frac{\bar{\gamma}}{2}\left(\frac{2\,\bar{\gamma}}{\bar{\gamma} + 1}\right)^{\frac{\bar{\gamma} + 1}{\bar{\gamma} - 1}} \left[\frac{p_{\max}\, Ax_m}{(1/2)\, W v_m^{\,2}}\right] V_{m1}^{\,2} \quad . \tag{8.10}$$

Define the two parameters,

$$\Gamma = \frac{\bar{\gamma}}{2}\left(\frac{2\bar{\gamma}}{\bar{\gamma}+1}\right)^{\frac{\bar{\gamma}+1}{\bar{\gamma}-1}} \tag{8.11}$$

and

$$E = \frac{(1/2)\,W v_m^2}{p_{max}\,A x_m} \; .^{1} \tag{8.12}$$

[1]Piezometric efficiency is defined by corner (d) as the ratio of the mean pressure to the maximum pressure. The mean pressure in this definition is the pressure which, when applied to the accelerated mass over the total stroke in the device, will produce the observed velocity. The maximum pressure is the observed maximum pressure.

Equation (8.12) may be derived as follows:

The kinetic energy, E_k, of the mass at the end of stroke is

$$E_k = 1/2\ W v_m^2$$

and, by the above definition of mean pressure, $\bar{p}$,

$$E_k = x_m A \bar{p} \; .$$

Equating the two expressions for E_k and solving for $\bar{p}$,

$$\bar{p} = \frac{W v_m^2}{2 x_m A} \; .$$

Dividing by p_{max}

$$E = \frac{\bar{p}}{p_{max}} = \frac{W v_m^2}{2 x_m A\, p_{max}} \; .$$

The ratio $\bar{p}/p_{max}$ gives an indication of the flatness of the pressure-travel curve. In a device with no retarding forces (a completely frictionless device accelerating a mass horizontally), the ratio would give a true indication of the flatness of the pressure-travel curve. As retarding forces increase in the device, the ratio indicates less certainly the shape of the pressure-travel curve. In the case, for example, where the retarding forces equaled the force accelerating the mass, a piezometric efficiency of only 50% would be obtained from a pressure-displacement curve having a constant pressure throughout the stroke.

Equation (8.10) then reduces to

$$X_m = \frac{\Gamma}{E} V_{m1}^{\,2} \ . \tag{8.13}$$

But X_m is also related to V_{m1} by equation (8.5). Hence, for any given value of E, V_{m1} can be determined from the implicit relationship

$$\frac{\Gamma}{E} V_{m1}^{\,2} = \left(1 - \frac{\bar{\gamma} - 1}{2} V_{m1}\right)^{-\frac{2}{\bar{\gamma} - 1}} - 1 \ . \tag{8.14}$$

X_m, C_b, and L_b may then be obtained from equations (8.5), (8.2), and (8.4), respectively.

There are two real roots of equation (8.14) corresponding to each value of E and $\bar{\gamma}$; however, the larger of the two roots generally leads to prohibitively large values of C_b and L_b and may be disregarded. Appendix E treats the cases in which the larger root must be used. In Figure 7a, we have plotted the smaller root, which is denoted by $\widetilde{V}_{m1}$ versus E for several values of $\bar{\gamma}$. The curves labeled E_{max} and E_{min} are such that:

if $E > E_{max}$, the roots of equation (8.14) are imaginary;

if $E < E_{min}$, the smaller root, $\widetilde{V}_{m1}$ is less than $1/\bar{\gamma} = V_M$ and condition b. is violated.

The problem of determining C_b and L_b to produce a given value of $E = f(p_{max}, v_m)$ is said to have a "feasible solution" if $\widetilde{V}_{m1}$ satisfies conditions a. and b., above, i. e.,

$$V_b \geq \widetilde{V}_{m1} \geq 1/\bar{\gamma} \ .$$

The procedure for determining the charge weight and web size may now be outlined.

Step 1

a. Given the maximum pressure p^*_{max} and the ejection velocity v^*_m together with the other ballistic parameters for a particular firing, calculate the values:

$$K_3 = 2\,gx_m \sin\theta / v_m^{*2}$$

$$K_4 = \frac{\phi^*_{max}\,(\eta - \frac{1}{\rho})}{24\,F}$$

$$\phi_{max} = \phi^*_{max} \Big/ \left(1 + \frac{K_3}{3 - 2n}\right)\left(1 + \frac{2K_4}{3 - 2n}\right)$$

$$v_m = v_m^* \Big/ \left(1 - \frac{(1-n)}{3 - 2n}K_3\right)\left(1 + \frac{K_4}{3 - 2n}\right)$$

$$W = (1 + K_1)\,w/g_c$$

$$\bar{\gamma} = \left(\frac{1 + K_1 + K_2}{1 + K_1}\right)(\gamma - 1) + 1$$

and

$$E = (1/2)\,W\,v_m^2 / \phi_{max} A x_m \;.$$

b. Determine from Figure 7a if E lies in the range $E_{min} \leq E \leq E_{max}$. If so, go on to Step 2; if not, use the method of Appendix E.

<u>Step 2</u>

a. Determine the values:

$\alpha = f(\phi_{max},\ n)$ from Figure 1

$\widetilde{V}_{m1} = f(E,\ \bar{\gamma})$ from Figure 7a

and

$X_m = f(\widetilde{V}_{m1},\ \bar{\gamma})$ from Figure 7.

b. Calculate the values

$U_i = 12\,Ax_m / X_m$

and

$$\lambda = \frac{A}{\alpha\,BF}\,v_m / \widetilde{V}_{m1} \;.$$

Step 3

Calculate the values

$$L_{b\,\min} = \frac{\alpha BW}{A} v_m$$

$$C_b = \rho(U_c - U_i), \text{ and}$$

$$L_b = C_b/\lambda .$$

The foregoing values give the required charge weight and web size provided $L_b \geq L_{b\,\min}$. If $L_b < L_{b\,\min}$ the problem has no feasible solution.

Sample Problem 3.

Other data being the same as in Sample Problem 2, what web and charge weight are required to yield $p^*_{max} = 6000$ psi, and $v^*_m = 10.5$ ft/sec? The solution is shown in Table 3. It is concluded that a constant-surface charge weighing 0.054 lb_m with a web of 0.73 in will meet the specifications. Notice, however, that the design leaves quite a bit of unburned propellant at ejection. The distance burned through the grain at the end of the stroke is only 0.30 in, i. e., about 41 per cent of the web. A discussion of this problem is given in Section IX, and Appendix E.

B. Given v_m

Frequently, the ejection velocity is specified without attaching much importance to the maximum pressure provided, of course, it stays within certain safety limits. In such cases several values of E may be selected in the permissable range and the corresponding maximum pressure determined from

$$p_{max} = \frac{(1/2)\, W\, v_m^2}{E\, Ax_m} .$$

Values of C_b and L_b may then be calculated in the usual way. This slight revision of the method has two advantages:

1. For any selected value of E it can be determined immediately whether or not a feasible solution exists, and

Table 3 DETERMINATION OF CHARGE WEIGHT AND WEB SIZE TO MEET GIVEN PERFORMANCE SPECIFICATIONS

Sample Problem 3.

Given Values

$A = 0.7854 \text{ in}^2$

$B = 0.01741 \ (\text{in}^2/\text{lb}_f)^n \text{ in/sec}$

$F = 3.623^5 \text{ ft.lb}_f/\text{lb}_m$

$K_1 = 0.10$

$K_2 = 0.25$

$n = 0.51$

$p^*_{max} = 6000 \text{ lb}_f/\text{in}^2$

$U_c = 6 \text{ in}^3$

$v^*_m = 10.50 \text{ ft/sec}$

$w = 2100 \text{ lb}_m$

$x_m = 1 \text{ ft}$

$\gamma = 1.215$

$\theta = 0$ degrees

$\sin\theta = 0$

$\rho = 0.06146 \text{ lb}_m/\text{in}^3$

$\eta = 30 \text{ in}^3/\text{lb}_m$

(Note: a^b denotes $a \times 10^b$)

Step 1

$K_3 = 2 g x_m \sin\theta / v_m^{*2} = 0$

$K_4 = \dfrac{p^*_{max}\left(\eta - \frac{1}{\rho}\right)}{24F} = 0.0095$

$p_{max} = p^*_{max} \Big/ \left(1 + \dfrac{K_3}{3-2n}\right)\left(1 + \dfrac{2K_4}{3-2n}\right) = 5943$

$v_m = v^*_m \Big/ \left(1 - \dfrac{(1-n)K_3}{3-2n}\right)\left(1 + \dfrac{K_4}{3-2n}\right) = 10.45$

$W = (1 + K_1)\, w/g_c = 71.797$

$\bar{\gamma} = \left(\dfrac{1 + K_1 + K_2}{1 + K_1}\right)(\gamma - 1) + 1 = 1.2639$

$E = (1/2)\, W v_m^2 / p_{max} A x_m = 0.8398$

Step 2

$\alpha = f(p_{max}, n)$; Figure 1 $= 1.82^{-2}$

$\widetilde{V}_{m1} = f(E, \bar{\gamma})$; Figure 7a $= 0.97$

$X_m = f(\widetilde{V}_{m1}, \bar{\gamma})$; Figure 2 $= 1.84$

$U_i = 12\, A x_m / X_m = 5.122$

$\lambda = \left(\dfrac{A}{\alpha B F}\right) v_m \widetilde{V}_{m1} = 0.0737$

Step 3

$L_{b\,min} = \dfrac{\alpha B W}{A} v_m = 0.3027 \text{ in}$

$C_b = \rho (U_c - U_i) = 0.05396 \text{ lb}_m$

$L_b = C_b/\lambda = 0.732 \text{ in}$

3. A wide range of permissable values of the web size and charge weight may be determined which yield very nearly the same ϕ_{max} and v_m.

Using the inequalities

$$V_b \geq \widetilde{V}_{m1} \geq V_M$$

it can be shown that for any selected value of E, a feasible solution exists if and only if v_m satisfies the relationship

$$\frac{(1/2)\, W\, v_m^2}{F\, \rho\, U_c} \leq \frac{\widetilde{V}_{m1}}{2} \left[1 - \left(\frac{12\, A x_m}{U_c} \right) (1/X_m) \right]$$

where

$$X_m = \left[1 - \frac{\widetilde{\gamma} - 1}{2}\, \widetilde{V}_{m1} \right]^{-\frac{2}{\widetilde{\gamma} - 1}} - 1 .$$

(Note that Figures 7a and 2 may be used to evaluate the right-hand member of the inequality; also if v_m satisfies the inequality for a particular value of E it will satisfy it for all larger values of E since the right-hand member is a monotonically increasing function of E.)

The alternative method may be summarized as follows.

<u>Step 1</u>

Given the ejection velocity, v_m^*, and other ballistic data, calculate

$$K_3 = \frac{2\, g x_m \sin\theta}{v_m^{*2}}$$

$$v_m = v_m^* \Big/ \left(1 - \frac{1 - n}{3 - 2n}\, K_3 \right) \quad [1]$$

$$W = (1 + K_1)\, w/g_c$$

[1] The covolume correction factor unduly complicates the method and so is not included.

$$\bar{\gamma} = \left(\frac{1 + K_1 + K_2}{1 + K_1}\right) (\gamma - 1) + 1,$$

and

$$E_1 = \frac{(1/2)\, W\, v_m^2}{F\, \rho\, U_c} .$$

Step 2

a. Select a value of E in the range $E_{min} \le E \le E_{max}$ from Figure 7a.

b. Determine

$$\widetilde{V}_{m1} = f(E, \bar{\gamma}) \text{ from Figure 7a and}$$

$$X_m = f(\widetilde{V}_{m1}, \bar{\gamma}) \text{ from Figure 2.}$$

c. Calculate

$$E_2 = \frac{\widetilde{V}_{m1}}{2} \left[1 - \left(\frac{12\, Ax_m}{U_c} \right)^{(1/X_m)} \right] .$$

If $E_1 \le E_2$ go on to Step 3. If not, select a larger value of E and repeat Step 2.

Step 3

a. Calculate $p_{max} = \dfrac{(1/2)\, W\, v_m^2}{E\, Ax_m}$.

b. Determine $\alpha = f(p_{max}, n)$ from Figure 1 .

c. Calculate the values

$$U_i = 12\, Ax_m / X_m$$

$$\lambda = \frac{A}{\alpha\, B\, F}\, v_m / \widetilde{V}_{m1}$$

$$C_b = \rho(U_c - U_i)$$

and

$$L_b = C_b / \lambda .$$

d. Repeat Steps 2 and 3 for several different values of E. Plot C_b versus L_b on one or two cycle log paper. The resulting curve gives the required charge weight as a function of the web size to yield the given ejection velocity.

The results of a typical calculation are plotted in Figure 8. Notice the insensitivity of p^*_{max} to large changes in C_b and L_b. In the pressure range $1600 \leq p^*_{max} \leq 1650$ (a variation of about 3 per cent) the web and charge weight vary by 350 per cent! This is a phenomenon common to most CAD's where low loading densities, i. e., $C_b / \rho U_c << 1$, are the rule.

C. <u>Given p_{max}</u>

A method may also be devised to handle the case of a given maximum pressure, with an unspecified ejection velocity. The method is only a slight revision of the one given in part B of this section.

<u>Step 1</u>

Given the maximum pressure, p^*_{max}, and other ballistic data, calculate

$$K_4 = \frac{p^*_{max}\left(\eta - \frac{1}{\rho}\right)}{24\ F}$$

$$p_{max} = p^*_{max} \bigg/ \left(1 + \frac{2K_4}{3 - 2n}\right)$$

$$W = (1 + K_1) w / g_c$$

and

$$\bar{\gamma} = \left(\frac{1 + K_1 + K_2}{1 + K_1}\right) (\gamma - 1) + 1 \; .$$

<u>Step 2</u>

a. Select a value of E in the range $E_{min} \leq E \leq E_{max}$ from Figure 7a.

b. Determine

$\widetilde{V}_{m1} = f(E, \bar{\gamma})$ from Figure 7a, and

$X_m = f(\widetilde{V}_{m1}, \bar{\gamma})$ from Figure 2.

c. Calculate

$$E_2 = \frac{\widetilde{V}_{m1}}{2} \left[1 - \left(\frac{12\ Ax_m}{U_c} \right) (1/X_m) \right]$$

and

$$E_3 = \frac{\phi_{max}\ Ax_m\ E}{F\ \rho\ U_c} .$$

d. If $E_3 \leq E_2$ go on to Step 3. If not, select a larger value of E and repeat Step 2.

Step 3

a. Calculate $v_m = \sqrt{\frac{\phi_{max}\ Ax_m\ E}{(1/2)\ W}}$.

b. Determine $\alpha = f(\phi_{max}, n)$ from Figure 1.

c. Calculate the values

$$U_i = 12\ Ax_m / X_m$$

$$\lambda = \frac{A}{\alpha\ BF}\ v_m / \widetilde{V}_{m1}$$

$$C_b = \rho(U_c - U_i)$$

and

$$L_b = C_b/\lambda .$$

d. Repeat Steps 2 and 3 for several values of E. A plot of C_b versus L_b on one or two cycle log paper gives the required charge weight versus web size to yield the given maximum pressure.

IX. BALLISTIC EFFECTS OF SMALL PARAMETER CHANGES

A useful outgrowth of the equations summarized in Section V is the fact that if one or more of the loading conditions of a particular CAD is changed, the corresponding change in the performance may be calculated and vice versa. This fact is particularly useful in modifying grain designs that do not quite meet performance specifications. For example, suppose the specifications for

a CAD call for a maximum pressure of 6000 psi. Using the method of Section VIII a charge weight, C_{b1}, and a web, L_{b1}, are determined which should meet the specifications. A charge is made up having the indicated dimensions and is fired in the device. The measured maximum pressure is 5000 psi. What changes in the grain design should be made to meet the specifications?

Assuming that burning takes place throughout the greater part of the stroke length, and that a mathematical pressure maximum is attained, equations (5.8), (5.9) and (3.9) permit us to write

$$\frac{U_c - C_{b1}/\rho}{(C_{b1}/L_{b1})^2} (5000)^{3-2n} = \frac{U_c - C_{b2}/\rho}{(C_{b2}/L_{b2})^2} (6000)^{3-2n} \tag{9.1}$$

where C_{b2} and L_{b2} are the new values of the charge weight and web size required to attain 6000 psi. More generally, if it is desired to change the maximum pressure, $p_{max\ 1}$, to a new value $p_{max\ 2} = (1+\epsilon)p_{max\ 1}$ where $|\epsilon| \ll 1$ we may write

$$\frac{U_c - C_{b1}/\rho}{(C_{b1}/L_{b1})^2} = \frac{U_c - C_{b2}/\rho}{(C_{b2}/L_{b2})^2} (1+\epsilon)^{3-2n} . \tag{9.2}$$

There are then three possible modifications of the charge design. (1) Keep the same surface area and increase the charge weight. In this case

$$\rho S = \frac{C_{b1}}{L_{b1}} = \frac{C_{b2}}{L_{b2}} \tag{9.3}$$

so that substitution in equation (9.2) yields

$$\rho U_c - C_{b1} = (\rho U_c - C_{b2})(1+\epsilon)^{3-2n} . \tag{9.4}$$

Defining Δ, the loading density, by

$$\Delta = \frac{C_{b1}}{\rho U_c} . \tag{9.5}$$

Equation (9.4) may be solved for C_{b2} to yield

$$C_{b2} = C_{b1}\left[\frac{1}{\Delta} - \left(\frac{1}{\Delta} - 1\right)(1+\epsilon)^{-(3-2n)}\right]$$

or approximately

$$C_{b2} \approx \left[1 + (3-2n)\ \frac{1-\Delta}{\Delta}\ \epsilon\right] C_{b1} \ . \tag{9.6}$$

From equation (9.3) we also obtain

$$L_{b2} = \left(\frac{C_{b2}}{C_{b1}}\right) L_{b1} \ . \tag{9.7}$$

Equations (9.6) and (9.7) then determine the new values of the charge weight and web size. (Notice that this modification is practical only if $\Delta >> 0$; otherwise the equations may lead to very large values of C_{b2} and L_{b2}.)

(2) Keep the same web and increase the charge weight.

In this case we may substitute into equation (9.2)

$$L_{b2} = L_{b1} \tag{9.8}$$

to obtain

$$\frac{\rho U_c - C_{b1}}{{C_{b1}}^2} = \frac{\rho U_c - C_{b2}}{{C_{b2}}^2}\ (1 + \epsilon)^{3-2n} \ . \tag{9.9}$$

Solving for C_{b2} then gives approximately

$$C_{b2} \approx \left(\frac{2 + (3 - 2n)\epsilon - \Delta}{2 + (3 - 2n)\epsilon\Delta - \Delta}\right) C_{b1} \ . \tag{9.10}$$

(3) Keep the same charge weight and decrease the web.

In this case

$$C_{b2} = C_{b1} \tag{9.11}$$

and equation (9.2) reduces approximately to

$$L_{b2} \approx \left(1 - \frac{3 - 2n}{2}\ \epsilon\right) L_{b1} \ . \tag{9.12}$$

In a similar manner, the variations in maximum pressure and ejection velocity due to changes in the other ballistic parameters may be determined. The following table summarizes the results of such an analysis.

Table 4 BALLISTIC VARIATIONS DUE TO SMALL PARAMETER CHANGES

1 Per Cent Increase in:	Per Cent Variation in Maximum Pressure	Per Cent Variation in Ejection Velocity
A	$-\left(\frac{2}{3-2n}\right)$	$\frac{2(1-n)}{3-2n} - \frac{1}{1+\delta}$
B	$\frac{2}{3-2n}$	$\frac{1}{3-2n}$
C_b	$\left(\frac{1}{3-2n}\right)\left(2+\frac{\Delta}{1-\Delta}\right)$	$\frac{1}{3-2n} + \left(\frac{\Delta}{1-\Delta}\right)\left(\frac{\delta}{1+\delta} - \frac{1-n}{3-2n}\right)$
F	$\frac{2}{3-2n}$	$\frac{1}{3-2n}$
L_b	$-\left(\frac{2}{3-2n}\right)$	$-\left(\frac{1}{3-2n}\right)$
U_c	$-\left(\frac{1}{3-2n}\right)\left(\frac{1}{1-\Delta}\right)$	$\left(\frac{1}{1-\Delta}\right)\left(\frac{1-n}{3-2n} - \frac{\Delta\,\delta}{1+\delta}\right)$
w	$\frac{1}{3-2n}$	$-\left(\frac{1-n}{3-2n}\right)$
x_m	0	$\frac{\delta}{1+\delta}$

Note:

$$\Delta = \frac{C_b}{\rho U_c}, \text{ and } \delta = \frac{U_i}{12\, Ax_m}$$

It should be emphasized once again that the determination of the variation effects is based on the assumptions: $C = \lambda L$, and $V_M \le V_{m1} \le V_b$. However, for small changes in the parameters the indicated variations in maximum pressure and ejection velocity should be of the right order of magnitude for most CAD firings.

Another useful relationship may be derived from equation (9.2). If the ratio, β, defined by

$$\beta \equiv \frac{U_c - C_b/\rho}{(C_b/L_b)^2}$$

is held constant, the maximum pressure is unchanged provided $V_\mu \neq V_b$. Furthermore for low loading densities, i. e., $\frac{C_b}{\rho U_c} << 1$, the ejection velocity is essentially unchanged for constant β. (This follows from equations (5.2), (5.5), and (5.3).) This relationship is useful when it is desirable to attain a given maximum pressure (or ejection velocity) and to have "burnt" occur at the moment of completion of the stroke. To achieve that end the following procedure may be used.

A value of v_m (or p_{max}) is assumed that leads to a value of E in the range $E_{min} \leq E \leq E_{max}$. U_i and λ are calculated using one of the methods in Section VIII. The values

$$\beta = \frac{U_c - C_b/\rho}{(C_b/L_b)^2} = \frac{U_i}{\lambda^2} \tag{9.13}$$

and

$$L_{b\ min} = \left(\frac{\alpha BW}{A}\right) v_m \tag{9.14}$$

are calculated. $L_{b\ min}$ is then the required web size. Solving for C_b in equation (9.13) then yields the required charge weight.

$$C_b \doteq \frac{L_b^2{}_{min}}{2\rho\beta}\left[\left(1 + \frac{4\beta\rho^2 U_c}{L_b^2{}_{min}}\right)^{1/2} - 1\right] . \tag{9.15}$$

X. DIMENSIONLESS SOLUTION; $C = \lambda_1 L + \lambda_2 L^2$

Similar solutions to those described in equations (5.5) through (5.14) may be obtained if the linear form function $C = \lambda L$ is replaced by a quadratic form function of the type

$$C = \lambda_1 L + \lambda_2 L^2 \quad . \tag{10.1}$$

Before elaborating further, let us evaluate the coefficients λ_1 and λ_2 in the equation. As before, let $\lambda = C_b/L_b$. We then obtain

$$\lambda = \lambda_1 + \lambda_2 L_b$$

or

$$\lambda_2 = \frac{\lambda - \lambda_1}{L_b} . \qquad (10.2)$$

Furthermore, since

$$\frac{dC}{dL} = \rho S \qquad (10.3)$$

where S is the burning surface at any instant, we also have

$$\frac{d^2C}{dL^2} = \rho \frac{dS}{dL} = 2\lambda_2 . \qquad (10.4)$$

Replacing $\frac{dS}{dL}$ with an average value, $\frac{(S_b - S_0)}{L_b}$, where S_0 and S_b are the values of S at $L = 0$ and $L = L_b$, respectively, and substituting into equation (10.2) yields

$$\lambda_1 = \lambda - \rho(S_b - S_0)/2 . \qquad (10.5)$$

Equations (10.2) and (10.5) are then the values of λ_1 and λ_2 to be used in equation (10.1). (As a matter of interest, it can be shown using the above equations, that if the true form function is a cubic of the type

$$C = \Lambda_1 L + \Lambda_2 L^2 + \Lambda_3 L^3$$

the values of λ_1 and λ_2 are given by

$$\lambda_1 = \Lambda_1 - (1/2)\Lambda_3 L_b^2$$

and

$$\lambda_2 = \Lambda_2 + (3/2)\Lambda_3 L_b .)$$

Values of λ_1 for some of the more common grain shapes are given in Appendix A. Equation (10.2) may then be used to determine λ_2.

After burnt, the ballistics are independent of the form function, hence we need only concern ourselves with the period before burnt. During the burning regime the system to be solved is

$$pA = W v \frac{dv}{dx} \qquad (10.6)$$

$$p \left[\frac{12\ Ax + U_i}{C} \right] = 12\ NRT \tag{10.7}$$

$$C_v C (T_v - T) = \left(\frac{1 + K_1 + K_2}{1 + K_1} \right) \frac{W}{2}\ v^2 \tag{10.8}$$

$$\frac{dL}{dt} = \alpha\, Bp \tag{10.9}$$

and

$$C = \lambda_1 L + \lambda_2 L^2 \ . \tag{10.10}$$

Combining equations (10.6) and (10.9) we again obtain the relation

$$L = \frac{\alpha\, BW}{A}\, v \ . \tag{10.11}$$

The system may then be reduced to the single differential equation

$$(12\ Ax + U_i)\ \frac{dv}{dx} = 12\ \alpha\, BF\, \lambda_1 + 12\ (\alpha\, B)^2\ \frac{F\, W \lambda_2}{A}\ v$$

$$- 6 \left(\frac{1 + K_1 + K_2}{1 + K_1} \right) (\gamma - 1)\ Av \ . \tag{10.12}$$

Defining $\bar{\gamma}$ and X as before,

$$\bar{\gamma} \equiv 1 + \left(\frac{1 + K_1 + K_2}{1 + K_1} \right) (\gamma - 1) \tag{10.13}$$

and

$$X \equiv \frac{12\ A}{U_i}\, x \tag{10.14}$$

but with a new velocity variable, V', defined as

$$V' \equiv \frac{A}{\alpha\, BF \lambda_1}\ v \equiv \frac{\lambda}{\lambda_1}\ V \tag{10.15}$$

equation (10.12) simplifies to

$$(1 + X)\ \frac{dV'}{dX} = 1 - \left[\frac{\bar{\gamma} - 1}{2} - \left(\frac{\alpha\, B}{A} \right)^2\ F\, W \lambda_2 \right] V' \ . \tag{10.16}$$

Now, employing equations (10.11) and (10.15), the value of V' at burnt is

$$V_b' = \frac{A}{\alpha BF\lambda_1} v_b = \left(\frac{A}{\alpha B}\right)^2 \frac{L_b}{F\lambda_1 W} . \tag{10.17}$$

Combining equations (10.16), (10.17), and (10.2) then yields

$$(1 + X) \frac{dV'}{dX} = 1 - \left[\frac{\bar{\gamma} - 1}{2} - \frac{\lambda - \lambda_1}{\lambda_1 V_b'}\right] V' . \tag{10.18}$$

If a new parameter, $\bar{\gamma}'$, is defined as

$$\bar{\gamma}' = \bar{\gamma} - \frac{2(\lambda - \lambda_1)}{\lambda_1 V_b'} \tag{10.19}$$

equation (10.18) finally reduces to

$$(1 + X) \frac{dV'}{dX} = 1 - \frac{\bar{\gamma}' - 1}{2} V' \tag{10.20}$$

which very conveniently has the same form as equation (4.6) with V' replacing V, and $\bar{\gamma}'$ replacing $\bar{\gamma}$. Separating variables and integrating, X is obtained as a function of V'

$$X = \left(\frac{1}{1 - \frac{\bar{\gamma}' - 1}{2} V'}\right)^{\frac{2}{\bar{\gamma}' - 1}} - 1 \tag{10.21}$$

which, of course, has the same form as equation (4.7).

Similarly, if a new pressure variable, P', is defined as

$$P' = \frac{U_i}{12W} \left(\frac{A}{\alpha BF\lambda_1}\right)^2 p = \left(\frac{\lambda}{\lambda_1}\right)^2 P \tag{10.22}$$

the relationship between P', V', and $\bar{\gamma}'$ is found to be

$$P' = V' \frac{dV'}{dX} = V' \left(1 - \frac{\bar{\gamma}' - 1}{2} V'\right)^{\frac{\bar{\gamma}' + 1}{\bar{\gamma}' - 1}} \tag{10.23}$$

which has the same form as equation (4.9). The value of V' at which P' attains its mathematical maximum, P_M', is thus

$$V_M' = \frac{1}{\bar{\gamma}'} \tag{10.24}$$

and

$$P'_M = V'_M \left(1 - \frac{\bar{\gamma}' - 1}{2} V'_M\right)^{\frac{\bar{\gamma}' + 1}{\bar{\gamma}' - 1}} = \frac{1}{\bar{\gamma}'}\left(\frac{\bar{\gamma}' + 1}{2\bar{\gamma}'}\right)^{\frac{\bar{\gamma}' + 1}{\bar{\gamma}' - 1}} . \qquad (10.25)$$

The value of T is found by substituting the new dimensionless parameters into equation (10.8). Thus,

$$T = \left(\frac{1 - \frac{\bar{\gamma}' - 1}{2} V'}{1 - \frac{\bar{\gamma}' - \bar{\gamma}}{2} V'}\right) T_v \qquad (10.26)$$

which, although not in the same form as equation (4.16), reduces to that equation when $\bar{\gamma}' = \bar{\gamma}$.

A special case, which till now has not been considered, occurs when $\bar{\gamma}' = 1$. Although the value of $\bar{\gamma}$ is always greater than one, $\bar{\gamma}'$ is not so restricted. (We shall, however, assume $\bar{\gamma}' > 0$ since otherwise the graphs become quite unwieldy.) Letting $\bar{\gamma}' = 1$, equation (10.20) becomes

$$\frac{dV'}{dX} = \frac{1}{1 + X} \qquad (10.27)$$

so that V' and X are related by the expression

$$V' = ln\ (1 + X) \qquad (10.28)$$

and, P' and V' by the expression

$$P' = V' \frac{dV'}{dX} = \frac{V'}{e^{V'}} . \qquad (10.29)$$

The solution is now complete. Figures 2 and 3 which relate the variables X, V, P, and $\bar{\gamma}$ for the case $C = \lambda L$, may be used to relate the new variables X, V', P', and $\bar{\gamma}'$ for the case $C = \lambda_1 L + \lambda_2 L^2$. Values of $\bar{\gamma}'$ need not be greater than one, hence the inclusion of values of $\bar{\gamma} \leq 1$ in the figures.

XI. GRAPHICAL DETERMINATION OF MAXIMUM PRESSURE AND EJECTION VELOCITY; $C = \lambda_1 L + \lambda_2 L^2$

The determination of maximum pressure and ejection velocity for the case of a quadratic form function is considerably more involved than that for a linear case. This is not surprising since $\bar{\gamma}'$, unlike the constant $\bar{\gamma}$,

is a function of the maximum pressure. However, in the case of extremely regressive or progressive grain configurations, the method developed here should lead to more accurate results than does the method of Section VII.

The following parameters are defined similarly to those in Section VII:

$$q'_1 \equiv \frac{U_i}{12\,W}\left(\frac{A}{BF\lambda_1}\right)^2 \equiv \left(\frac{\lambda}{\lambda_1}\right)^2 q_1 \tag{11.1}$$

$$q'_2 \equiv \frac{A}{BF\,\lambda_1} \equiv \left(\frac{\lambda}{\lambda_1}\right) q_2 \tag{11.2}$$

$$q'_3 \equiv \left(\frac{A}{B}\right)^2 \quad \frac{L_b}{F\lambda_1 W} \equiv \left(\frac{\lambda}{\lambda_1}\right) q_3 \tag{11.3}$$

$$Q'_1 \equiv q'_1/\alpha^2 \tag{11.4}$$

and

$$Q'_2 \equiv q'_2/\alpha\,. \tag{11.5}$$

The dimensionless variables P', V', and V'_b are then written,

$$P' = Q'_1\,p = q'_1 p/\alpha^2 \tag{11.6}$$

$$V' = Q'_2\,v = q'_2 v/\alpha \tag{11.7}$$

and

$$V'_b = q'_3/\alpha^2\,. \tag{11.8}$$

Since $\frac{P'_\mu}{q'_1} = \frac{p_{max}}{\alpha^2} = \frac{P_\mu}{q_1}$, Figures 6a and 6b may be used to determine α as a function of n and the ratio $\frac{P'_\mu}{q'_1}$.

The procedure for determining maximum pressure and ejection velocity for a charge with a quadratic form function may now be outlined.

Step 1

a. Given the firing data calculate the values of the parameters:

$$\lambda = C_b / L_b$$

$$\lambda_1 = \lambda - \rho(S_b - S_o)/2$$

$$\bar{\gamma} = \left(\frac{1 + K_1 + K_2}{1 + K_1}\right)(\gamma - 1) + 1$$

$$U_i = U_c - C_b/\rho$$

$$W = (1 + K_1)\frac{w}{g_c}$$

$$q'_1 = \frac{U_i}{12W}\left(\frac{A}{BF\lambda_1}\right)^2$$

$$q'_2 = \frac{A}{BF\lambda_1}$$

$$q'_3 = \left(\frac{A}{B}\right)^2 \frac{L_b}{F\lambda_1 W} \text{ , and}$$

$$X_m = \frac{12\, Ax_m}{U_i} .$$

b. Assume $\bar{\gamma}' = \bar{\gamma}$.

c. Calculate $V'_M = \frac{1}{\bar{\gamma}'}$.

d. Determine $V'_{m1} = f\,(X_m, \bar{\gamma}')$ from Figure 2.

Step 2

a. Assume a value of P'_μ. (Initially, this is done by selecting the value of P' from Figure 3 corresponding to the smaller of the two values V'_{m1} and V'_M as determined in Step 1.)

b. Calculate P'_μ / q'_1 .

c. Determine $\alpha = f\left(n, \frac{P'_\mu}{q'_1}\right)$ from Figure 6.

d. Calculate the values

$$V'_b = q'_3/\alpha^2$$

$$\bar{\gamma}' = \bar{\gamma} - \frac{2(\lambda - \lambda_1)}{\lambda_1 V'_b} \text{ , and}$$

$$V'_N = \frac{1}{\bar{\gamma}'} \text{ .}$$

e. Determine $V'_{m1} = f(X_m, \bar{\gamma}')$ from Figure 2.

f. Let $V'_\mu = \min (V'_N, V'_{m1}, V'_b)$. Determine $P'_\mu = f(V'_\mu, \bar{\gamma}')$ from Figure 3.

g. If the assumed value of P'_μ equals the calculated value go on to Step 3. If not, repeat Step 2 using the calculated value of P'_μ.

Step 3

a. Calculate

$$Q'_1 = q'_1/\alpha^2 \text{ , and}$$

$$Q'_2 = q'_2/\alpha \text{ .}$$

b. Calculate the maximum pressure using $p_{max} = P'_\mu/Q'_1$.

Step 4

Case A. $V'_b \geq V'_{m1}$

The ejection velocity is then calculated from

$$v_m = V'_{m1}/Q'_2 \text{ .}$$

Case B. $V'_b < V'_{m1}$

a. Determine $X_b = f(V'_b, \bar{\gamma}')$ from Figure 2.

b. Calculate: $\nu = \dfrac{1 + X_b}{1 + X_m}$

$$\frac{\bar{\gamma} - 1}{2} V_b = \frac{\bar{\gamma} - 1}{2} \frac{\lambda_1}{\lambda} V'_b \text{ , and}$$

$$\frac{\bar{\gamma} - 1}{2} Q_2 = \frac{\bar{\gamma} - 1}{2} \frac{\lambda_1}{\lambda} Q'_2 \quad .$$

c. Determine: $\phi = f(\nu, \bar{\gamma})$ from Figure 4.

$$\frac{\bar{\gamma} - 1}{2} V_{m2} = f\left(\frac{\bar{\gamma} - 1}{2} V_b, \phi\right) \quad \text{from Figure 5.}$$

d. The ejection velocity is then

$$v_m = \frac{\frac{\bar{\gamma} - 1}{2} V_{m2}}{\frac{\bar{\gamma} - 1}{2} Q_2} \quad .$$

Step 5

The correction factors of Section VI may be applied if desired.

Calculate

$$K_3 = \frac{2\, g x_m \sin \phi}{v_m^2}$$

and,

$$K_4 = \frac{p_{\max} \left(\eta - \frac{1}{\rho}\right)}{24F} \quad .$$

The corrected values of maximum pressure and ejection velocity are then

$$p^*_{\max} = \left(1 + \frac{1}{3-2n} K_3\right) \left(1 + \frac{2}{3-2n} K_4\right) p_{\max}$$

and

$$v^*_m = \left(1 - \frac{1-n}{3-2n} K_3\right) \left(1 + \frac{1}{3-2n} K_4\right) v_m \; .$$

Sample Problem 4.

Given the following data,

$A = 4.92 \text{ in}^2$

$B = 0.01919 \; (\text{in}^2/\text{lb}_f)^n \; (\text{in/sec})$

$C_b = 0.297 \text{ lb}_m$

$F = 328{,}250$ ft lb_f/lb_m

$K_1 = 0.089$

$K_2 = 0.25$

$L_b = 0.06361$ in

$n = 0.372$

$S_o = 67.67$ in^2, $S_b = 97.28$ in^2.

$U_c = 159.1$ in^3

$w = 360$ lb_m

$x_m = 3.333$ ft

$\theta = 70.5°$

$\gamma = 1.24$

$\rho = 0.05661$ lb_m/in^3

$\eta = 30$ in^3/lb_m

determine the maximum pressure and ejection velocity.

The solution is shown in Table 5. The results, $p^*_{max} = 1684$ lb_f/in^2 and $v^*_m = 56.8$ ft/sec compare favorably with 1647 lb_f/in^2 and 61.6 ft/sec obtained from numerical integration. (Disregarding λ_2, and using the method of Section VII yields the results $p^*_{max} = 1930$ psi and $v^*_m = 62.1$ ft/sec.)

XII. DESIGN OF A SCALE MODEL TO REPRODUCE BALLISTICS OF A LARGER CAD

The dimensionless variables defined previously may be used to determine families of CAD's having identical pressure-distance and velocity-distance curves provided the distance scales are suitably adjusted. A very useful application of this feature is that it permits the determination of the dimensions of a small scale model to reproduce the ballistics of a larger parent CAD. The use of a small model as a testing apparatus could lead to considerable saving in expenditure.

Table 5

DETERMINATION OF MAXIMUM PRESSURE AND EJECTION VELOCITY; $C = \lambda_1 L + \lambda_2 L^2$

Sample Problem 4.

Given Values
$A = 4.92$ in^2
$B = 0.01919$ (in^2/lb$_f$)^n in/sec
$C_b = 0.297$ lb$_m$
$F = 328{,}250$ ft·lb$_f$/lb$_m$
$I_1 = 0.089$
$I_2 = 0.25$
$L_b = 0.06361$ in
$n = 0.372$
$S_o = 67.67$ in^2
$S_b = 97.28$ in^2
$U_c = 159.1$ in^3
$w = 360$ lb$_m$
$x_m = 3.333$ ft
$\theta = 70.5°$
$\gamma = 1.24$
$\rho = 0.05661$ lb$_m$/in^3
$\eta = 30$ in^3/lb$_m$

Step 1
$\lambda = C_b/L_b = 4.669$
$\lambda_1 = \lambda - \rho(S_b - S_o)/2 = 3.831$
$\bar{\gamma} = \left(\frac{1 + I_1 + I_2}{1 + I_1}\right)(\gamma - 1) + 1 = 1.295$
$U_i = U_c - C_b/\rho = 153.8$
$W = (1 + I_1)\frac{w}{U_c} = 12.19$
$q'_1 = \frac{U_i}{12W}\left(\frac{A}{BF\lambda_1}\right)^2 = 4.372^{-8}$
$q'_2 = \frac{A}{BF\lambda_1} = 2.039^{-4}$
$q'_3 = \left(\frac{A}{B}\right)^2 \frac{L_b}{F\lambda_1 W} = 2.729^{-4}$
$X_m = 12\,Ax_m/U_i = 1.280$
$\bar{\gamma}' = \bar{\gamma} = 1.295$
$V'_N = 1/\bar{\gamma}' = 0.7722$
$V'_{m1} = f(X_m, \bar{\gamma}')$; Figure 2 $= 0.78$

Step 2	Trial Numbers 1	2	3	
P'_μ =	0.2975	0.380	0.365	Assumed; Figure 3
P'_μ/q'_1 =	6.805^6	8.692^6	8.349^6	
α =	1.5^{-2}	1.4^{-2}	1.4^{-2}	$\alpha = f(n, P'_\mu/q'_1)$; Figure 6
α^2 =	2.25^{-4}	1.96^{-4}	1.96^{-4}	
V'_b =	1.213	1.392	1.392	$V'_b = q'_3/\alpha^2$
$\bar{\gamma}'$ =	0.9343	0.9807	0.9807	$\bar{\gamma}' = \bar{\gamma} - 2\left(\frac{\lambda - \lambda_1}{\lambda_1 V'_b}\right)$
V'_N =	1.0703	1.0197	1.0197	$V'_N = 1/\bar{\gamma}'$
V'_{m1} =	0.850	0.840	0.840	$V'_{m1} = f(X_m, \bar{\gamma}')$; Figure 2
V'_μ =	0.850	0.840	0.840	$V'_\mu = \min(V'_b, V'_N, V'_{m1})$
P'_μ =	0.380	0.365	0.365	$P'_\mu = f(V'_\mu, \bar{\gamma}')$; Figure 3

Step 3	
$Q'_1 = 2.231^{-4}$	$Q'_1 = q'_1/\alpha^2$
$Q'_2 = 1.456^{-2}$	$Q'_2 = q'_2/\alpha$
$\phi_{max} = 1636$	$\phi_{max} = P'_\mu/Q'_1$

Step 4 Case A: $V'_b \geq 0\,V'_{m1}$? $\boxed{\times}$ $v_m = V'_{m1}/Q'_2 = 57.7$

Case B: $V'_b < V'_{m1}$? $\square$

$X_b = f(V'_b, \bar{\gamma}')$; Figure 2 =

$\nu = (1 + X_b)/(1 + X_m) =$

$\frac{\bar{\gamma}-1}{2} V_b = \frac{\bar{\gamma}-1}{2}\frac{\lambda_1}{\lambda} V'_b =$

$\frac{\bar{\gamma}-1}{2} Q_2 = \frac{\bar{\gamma}-1}{2}\frac{\lambda_1}{\lambda} Q'_2 =$

$\phi = f(\nu, \bar{\gamma})$; Figure 4 =

$\frac{\bar{\gamma}-1}{2} V_{m2} = f\left(\frac{\bar{\gamma}-1}{2} V_b, \phi\right)$; Figure 5 =

$v_m = \frac{\bar{\gamma}-1}{2} V_{m2} \Big/ \frac{\bar{\gamma}-1}{2} Q_2 =$

Step 5

$$I_3 = \frac{2\,gx_m \sin\theta}{v_m^2} = 0.0607$$

$$I_4 = \frac{\phi_{max}\left(\eta - \frac{1}{\rho}\right)}{24\,F} = 0.0026$$

$$\phi^*_{max} = \left(1 + \frac{1}{3-2n} I_3\right)\left(1 + \frac{2}{3-2n} I_4\right)\phi_{max} = 1684 \text{ psi}$$

$$v^*_m = \left(1 - \frac{1-n}{3-2n} I_3\right)\left(1 + \frac{1}{3-2n} I_4\right) v_m = 56.8 \text{ ft/sec}$$

(Note: a^b denotes $a \times 10^b$)

The variables as defined in Section V are:

$$P = \frac{U_i}{12\,W}\left(\frac{A}{\alpha B F \lambda}\right)^2 p \tag{12.1}$$

$$V = \frac{A}{\alpha B F \lambda}\,v \tag{12.2}$$

$$X = \frac{12\,A}{U_i}\,x, \text{ and} \tag{12.3}$$

$$V_b = \left(\frac{A}{\alpha B}\right)^2 \frac{L_b}{F \lambda W} \;. \tag{12.4}$$

Let τ be a ballistic parameter which may be arbitrarily specified. The form of the solution equations given in Section V then indicates that all CAD's with

$$\frac{U_i}{W}\left(\frac{A}{\alpha B F \lambda}\right)^2 = k_1 \tag{12.5}$$

$$\frac{A}{\alpha B F \lambda} = k_2 \tag{12.6}$$

$$\frac{A\tau}{U_i} = k_3 \tag{12.7}$$

and

$$\left(\frac{A}{\alpha B}\right)^2 \frac{L_b}{F \lambda W} = k_4 \,, \tag{12.8}$$

where the k_i are constants, have identical pressures and velocities at the point x/τ. Moreover, if in addition

$$\frac{x_m}{\tau} = k_5 \tag{12.9}$$

all such devices have identical ejection velocities. We shall call those CAD's satisfying equations (12.5) through (12.9) "ballistically similar". If the same propellant composition is used, the requirements for ballistic similarity reduce to

$$x_m = \nu_1 \tau \tag{12.10}$$

$$L_b = \nu_2 \tau \tag{12.11}$$

$$C_b = \nu_3 A \tau \tag{12.12}$$

$$U_c = \nu_4 A \tau \tag{12.13}$$

$$w = \nu_5 A \tau \tag{12.14}$$

where, again, the ν_i are constants. Note also that equations (12.11) and (12.12) imply that the form function coefficients of ballistically similar devices must satisfy

$$\Lambda_1 = \nu_6 A \tag{12.15}$$

$$\Lambda_2 = \nu_7 A/\tau \text{, and} \tag{12.16}$$

$$\Lambda_3 = \nu_8 A/\tau^2 \tag{12.17}$$

since then and only then will the equation

$$C_b = \Lambda_1 L_b + \Lambda_2 L_b{}^2 + \Lambda_3 L_b{}^3$$

be valid.

The choice of the parameter τ is arbitrary within certain limits. It can be any function of the six variables A, x_m, L_b, C_b, U_c, and w provided the function is consistent dimensionally. For practical reasons, however, it should be a simple function so that given the value of one of the six variables, the values of the other five may be determined directly. As practical selections of τ we might have:

$$\tau = A$$

$$\tau = 2\sqrt{\frac{A}{\pi}} \text{ , (the piston diameter)}$$

or

$$\tau = 1/A \text{ .}$$

The following numerical example will help to illustrate the use of equations (12.10) - (12.14).

A CAD has the following dimensions:

$A = 4.92 \text{ in}^2$

$U_c = 159.1 \text{ in}^3$

$w = 300 \text{ lb}_m$

$X_m = 8.2 \text{ ft}$

$\theta = 0°$.

Its standard charge is composed of six uninhibited single-perf grains with dimensions

$$D = 0.51 \text{ in}$$

$$d = 0.146 \text{ in}$$

$$h = 3.126 \text{ in}$$

which lead to the form function coefficients

$$\Lambda_1 = 2.4541 \text{ lb}_m/\text{in}$$

$$\Lambda_2 = -1.4837 \text{ lb}_m/\text{in}^2$$

$$\Lambda_3 = 0 \text{ lb}_m/\text{in}^3 \ .$$

The propellant has the ballistic properties:

$$B = 0.009436 \text{ (in/sec)} \ (\text{in}^2/\text{lb}_f)^n$$

$$C_b = 0.21104 \text{ lb}_m$$

$$L_b = 0.091 \text{ in}$$

$$n = 0.58$$

$$T_v = 2410 \ {}^\circ K$$

$$\rho = 0.06 \text{ lb}_m/\text{in}^3$$

$$\eta = 30 \text{ in}^3/\text{lb}_m$$

$$\gamma = 1.239 \ .$$

A scale model is to be made with a cross-sectional area, A, of 1 in^2; what should be the other dimensions of the device and those of the propellant?

We shall solve the problem for two cases by specifying two different parameters for τ.

Case I: $\tau = A$

Substituting the value $\tau = A$ into equations (12.10) - (12.17) we have

$$x_m = \nu_1 A$$

$$L_b = \nu_2 A$$

$$C_b = \nu_3 A^2$$

$$U_c = \nu_4 A^2$$

$$w = \nu_5 A^2$$

$$\Lambda_1 = \nu_6 A$$

$$\Lambda_2 = \nu_7$$

$$\Lambda_3 = \nu_8 / A \quad .$$

We then calculate the values of the ν_i from the given dimensions of the parent CAD.

$$\nu_1 = x_m / A = 8.2/4.92 = 1.6667$$

$$\nu_2 = L_b / A = 0.091/4.92 = 0.018496$$

$$\nu_3 = C_b / A^2 = 0.21104/(4.92)^2 = 0.0087184$$

$$\nu_4 = U_c / A^2 = 159.1/(4.92)^2 = 6.5726$$

$$\nu_5 = W / A^2 = 300/(4.92)^2 = 12.3934$$

$$\nu_6 = \Lambda_1 / A = 2.4541/4.92 = 0.49880$$

$$\nu_7 = \Lambda_2 = -1.4837$$

$$\nu_8 = \Lambda_3 A = 0 \quad .$$

Setting $A = 1$, we then calculate the model dimensions:

$$x_m = \nu_1 A = (1.6667)(1) = 1.6667 \text{ ft}$$

$$L_b = \nu_2 A = 0.018496 \text{ in}$$

$$C_b = \nu_3 A^2 = 0.0087184 \text{ lb}_m$$

$$U_c = \nu_4 A^2 = 6.5726 \text{ in}^3$$

$$w = \nu_5 A^2 = 12.393 \text{ lb}_m$$

$$\Lambda_1 = \nu_6 A = 0.49880 \text{ lb}_m/\text{in}$$

$$\Lambda_2 = \nu_7 = -1.4837 \text{ lb}_m/\text{in}^2$$

$$\Lambda_3 = \nu_8 / A = 0 \text{ lb}_m/\text{in}^3 \quad .$$

If six single-perf grains are desired, the form function coefficients lead to the grain dimensions

$$D = 0.365 \text{ in}$$

$$d = 0.291 \text{ in}$$

$$h = 0.635 \text{ in} .$$ [1]

For purposes of comparison the dimensions of the two devices are listed in Table 6.

Table 6 COMPARISON OF DIMENSIONS OF PARENT CAD AND MODEL CAD USING $\tau = A$, THE PISTON CROSS-SECTIONAL AREA

	Parent CAD	Model CAD
A	4.92 in^2	1 in^2
C_b	0.21104 lb_m	0.00872 lb_m
L_b	0.091 in	0.0185 in
U_c	159.1 in^3	6.57 in^3
w	300 lb_m	12.39 lb_m
x_m	8.2 ft	1.6667 ft
N	6 grains	6 grains
D	0.51 in	0.365 in
d	0.146 in	0.291 in
h	3.126 in	0.635 in

[1] The grain dimensions are calculated from the formulas given in Appendix A, i. e.,

$$C_b = N \rho \frac{\pi}{4} (D^2 - d^2) h$$

$$\Lambda_1 = \frac{4C_b}{h(D-d)} \left[h + \frac{D-d}{2} \right]$$

$$\Lambda_2 = \frac{-8C_b}{h(D-d)}$$

for $N = 6$.

Using numerical integration, the ballistic performance of the two devices was calculated. Their pressure versus x/τ and velocity versus x/τ curves were found to be identical. In particular, they both yield p_{max} = 1403 psi and v_m = 101.2 ft/sec. The results are plotted in Figure 9.

Case II: $\tau = \Omega = 2\sqrt{\frac{A}{\pi}}$, the Piston Diameter

Substituting the value $\tau = \Omega$ into equations (12.10) - (12.17) we have

$$x_m = \nu_1 \Omega$$

$$L_b = \nu_2 \Omega$$

$$C_b = \nu_3 \Omega^3$$

$$U_c = \nu_4 \Omega^3$$

$$w = \nu_5 \Omega^3$$

$$\Lambda_1 = \nu_6 \Omega^2$$

$$\Lambda_2 = \nu_7 \Omega$$

$$\Lambda_3 = \nu_8 \quad .$$

Substituting the given dimensions of the parent CAD into the foregoing equations we then calculate the values of the ν_i.

$$\nu_1 = x_m/\Omega = 8.2/2.5029 = 3.2762$$

$$\nu_2 = L_b/\Omega = 0.091/2.5029 = 0.03636$$

$$\nu_3 = C_b/\Omega^3 = 0.21104/(2.5029)^3 = 0.01346$$

$$\nu_4 = U_c/\Omega^3 = 159.1/(2.5029)^3 = 0.01346$$

$$\nu_5 = w/\Omega^3 = 300/(2.5029)^3 = 10.147$$

$$\nu_6 = \Lambda_1/\Omega^2 = 2.4541/(2.5029)^2 = 0.3917$$

$$\nu_7 = \Lambda_2/\Omega = -1.4837/2.5029 = -0.5928$$

$$\nu_8 = \Lambda_3 \quad = 0$$

Setting $\Omega = 1.1284$ in, for which $A = 1$ in^2, the model dimensions are then calculated.

$$x_m = \nu_1 \Omega = (3.2762) \; (1.1284) = 3.697 \text{ ft}$$

$$L_b = \nu_2 \Omega = 0.04103 \text{ in}$$

$$C_b = \nu_3 \Omega^3 = 0.01934 \text{ lb}_m$$

$$U_c = \nu_4 \Omega^3 = 14.58 \text{ in}^3$$

$$w = \nu_5 \Omega^3 = 27.49 \text{ lb}_m$$

$$A_1 = \nu_6 \Omega^2 = 0.4988 \text{ lb}_m/\text{in}$$

$$A_2 = \nu_7 \Omega = -0.6689 \text{ lb}_m/\text{in}^2$$

$$A_3 = \nu_8 = 0 \text{ lb}_m/\text{in}^3$$

Taking $N = 6$ grains[1], the grain dimensions are calculated using the formulas on page 2 of Appendix A, to be

$$D = 0.230 \text{ in}$$

$$d = 0.0658 \text{ in}$$

$$h = 1.410 \text{ in} .$$

Using numerical integration techniques, the ballistic performance was calculated for the model device. As before, the parent CAD and the model CAD were found to have identical pressure and velocity versus x/τ curves. The comparative dimensions are listed in Table 7.

[1]As in the previous case, N may be set at any value for which D, d, and h, have positive values. If $N = 3$, for example, the dimensions are calculated to be

$$D = 0.378 \text{ in}$$

$$d = 0.214 \text{ in}$$

$$h = 1.410 \text{ in} .$$

The important thing to remember is that the charge configuration is not unique. Any configuration that has the form function,

$$C = 0.4988\, L - 0.6689\, L^2$$

and web

$$L_b = 0.04103 \text{ in}$$

will produce the same results.

Table 7

COMPARISON OF DIMENSIONS OF PARENT CAD AND MODEL CAD USING $\tau = \Omega$, THE PISTON DIAMETER

	Parent CAD	Model CAD
A	4.92 in^2	1 in^2
C_b	0.21104 lb$_m$	0.0193 lb$_m$
L_b	0.091 in	0.041 in
U_c	159.1 in^3	14.58 in^3
w	300 lb$_m$	27.49 lb$_m$
x_m	8.2 ft	3.70 ft
N	6	6
D	0.51 in	0.23 in
d	0.146 in	0.066 in
h	3.126 in	1.41 in

From the results of Cases I and II we conclude that it is possible to determine the dimensions of a small scale model that is ballistically similar to a larger parent CAD. The model could be used for experimental purposes. Once the proper charge weight and web size were determined for the model, the proper charge design for the parent device could be deduced from the results.

If, now, τ is taken to be x_m, all devices with equal ratios,

U_b/U_c, will satisfy equations (12.10) and (12.13).

For a particular device the remaining equations determine the charge design and accelerated mass. We can then say that for a given CAD, any device with the same ratio, U_b/U_c, may be used as an experimental model. The following numerical example illustrates the principle.

Device I has the following dimensions:

$$A = 4.92 \text{ in}^2$$
$$U_c = 159.1 \text{ in}^3$$
$$w = 300 \text{ lb}_m$$
$$x_m = 8.2 \text{ ft}$$
$$\theta = 0°.$$

The propellant has the properties:

$B = 0.009436 \text{ (in/sec) } (\text{in}^2/\text{lb}_f)^n$

$C_b = 0.21104 \text{ lb}_m$

$L_b = 0.091 \text{ in}$

$n = 0.58$

$T_v = 2410° K$

$\rho = 0.06 \text{ lb}_m/\text{in}^3$

$\eta = 30 \text{ in}^3/\text{lb}_m$

$\bar{\gamma} = 1.239$

and form function coefficients:

$\Lambda_1 = 2.4541 \text{ lb}_m/\text{in}$

$\Lambda_2 = -1.4837 \text{ lb}_m \text{in}^2$

$\Lambda_3 = 0 \text{ lb}_m/\text{in}^3$.

The resulting pressure and velocity versus x/x_m curves are shown in Figure 10. Two other devices have dimensions:

Device II:

$A = 1 \text{ in}^2$

$U_c = 3.944 \text{ in}^3$

$x_m = 1 \text{ ft}$

$\theta = 0°$

and

Device III:

$A = 2 \text{ in}^2$

$U_c = 23.661 \text{ in}^3$

$x_m = 3 \text{ ft}$

$\theta = 0°$.

(Note that $U_b/U_c = 3.043$ for the three devices.) Using the same type of propellant, what grain design and accelerated mass should be used in each device so that all three devices produce the curves shown in Figure 10?

Solution

Replacing τ by x_m in equations (12.10) - (12.17) we solve for the ν_i using the properties of Device I

$$\nu_2 = L_b/x_m = 0.01110$$

$$\nu_3 = C_b/Ax_m = 0.005231$$

$$\nu_5 = w/Ax_m = 7.4360$$

$$\nu_6 = \Lambda_1/A = 0.4988$$

$$\nu_7 = \Lambda_2 x_m/A = -2.4729$$

$$\nu_8 = \Lambda_3 x_m^2/A = 0,$$

from which we determine the values:

	Device II	Device III
$L_b = \nu_2 x_m$	0.01110 in	0.03330 in
$C_b = \nu_3 Ax_m$	0.005231 lb_m	0.031386 lb_m
$w = \nu_5 Ax_m$	7.4360 lb_m	44.6160 lb_m
$\Lambda_1 = \nu_6 A$	0.4988 lb_m/in	0.9976 lb_m/in
$\Lambda_2 = \nu_7 A/x_m$	-2.4729 lb_m/in^2	-1.6486 lb_m/in^2
$\Lambda_3 = \nu_8 A/x_m^2$	0 lb_m/in^3	0 lb_m/in^3
A	1 in^2	2 in^2
U_c	3.944 in^3	23.661 in^3
x_m	1 ft	3 ft

Integrating the ballistic equations numerically and plotting the results versus x/x_m for both devices produces the curves shown in Figure 10.

At this point some mention of an apparent relationship between the action times of ballistically similar devices should be made. As a conjecture we state that at equal values of x/τ all such devices have equal values of t/τ. The conjecture has not been proved, but there are strong arguments in its favor. Consider the following.

The previous derivations indicate that every member of a family of ballistically similar devices has the same velocity equation

$$\frac{dx}{dt} = f\left(\frac{x}{\tau}\right)$$

where $f(x/\tau)$ is a particular function of x/τ. Separating variables the equation may be integrated to yield

$$t = \tau \int_0^{x/\tau} \frac{dz}{f(z)}$$

The integral is improper and may not exist since $f(o) = 0$. Ignoring the fact, however, and assuming the existence of the integral the stated conjecture follows.

To substantiate the argument, in Figure 11 we have plotted the value t/τ versus x/τ for Devices I, II, and III, where $\tau = x_m$. (The data were obtained numerically.) Although there are actually slight differences between the curves they are undetectable from the figure and are possibly due to inherent errors in the numerical integration method. At any rate the evidence seems to indicate that the conjecture given above is at least a very good approximation to the real case.

The conjecture may be stated in another form: for ballistically similar devices the time required to reach the point x/τ is proportional to τ. Stated in this form it is easy to deduce a very useful application. From a given pressure-time trace for, say, Device D_1 we can determine the corresponding trace for any other ballistically similar Device D_2 simply by multiplying the abscissa values by the ratio τ_{D2}/τ_{D1}. The same is likewise true for pressure-distance, velocity-time, and velocity-distance curves.

In our discussion of ballistically similar devices we have neglected to consider non-horizontal firings. This has not been unintentional. A study of the non-horizontal equations was undertaken but, unfortunately, no simple similarity relationships could be found. However, the foregoing analyses seem to be approximately applicable to the non-horizontal case. Performance calculations were made for Devices I, II, and III assuming $\theta = 90°$. The results are plotted in Figure 12.

Although the velocity curves are very similar, the pressure and time curves vary by as much as 7%. This would indicate that in the case of non-horizontal firing the quations should be used with some discretion.

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APPENDIX A

FORM FUNCTIONS OF SOME COMMON GRAIN CONFIGURATIONS

The law of burning, known as Piobert's law, is generally adopted by interior ballisticians. According to the law the surface of each grain in the charge recedes parallel to itself as burning proceeds. Some confirmation that the law is nearly obeyed is obtained by firing charges such that the propellant is not all burned at the end of the stroke. The shape of each unburned grain is, in general, found to be very well preserved. In other words, simultaneous ignition of the surfaces of the grains and burning by parallel layers seem to be reasonable assumptions. (See Hunt (e) page 40.)

A grain whose surface increases as burning proceeds is called "progressive"; if the surface decreases it is called "regressive"; and if the surface remains constant it is called "neutral". Some of the more common grain shapes and their form functions are described here.

1. Cylindrical Grains

To obtain the form function of a charge made up of homogenecus cylindrical grains, the total charge weight, C_b, is equated to the number of grains, N, times the density of the propellant, ρ, times the volume of one grain. Thus,

$$C_b = N \rho \frac{\pi D^2}{4} h \qquad \text{(A1.1)}$$

where, D and h are, respectively, the initial diameter and the initial length of the grain. After burning through a distance, L, the charge left unburned is

$$C_b - C = N \rho \frac{\pi}{4} (D - 2L)^2 (h - 2L) . \qquad \text{(A1.2)}$$

Combining the two equations and rearranging terms, the form function is obtained as

$$C = \frac{2C_b}{D^2 h} \left[D(D + 2h)L - 2 (2D + h)L^2 + 4L^3 \right] . \qquad \text{(A1.3)}$$

The charge is regressive. The value of λ_1 to be used with a quadratic form function is

$$\lambda_1 = \frac{C_b}{L_b} - \frac{\rho(S_b - S_o)}{2} = \frac{2C_b}{Dh} \left[D + 2h - \frac{2{L_b}^2}{D} \right] . \qquad \text{(A1.4)}$$

If $D \geq h$, the web is given by

$$L_b = \frac{h}{2} \ . \tag{A1.5}$$

If $D < h$, the web is given by

$$L_b = \frac{D}{2} \ . \tag{A1.6}$$

A common variant of the grain design is to "inhibit" the cylindrical surface so that burning takes place only on the circular ends of the grains. A neutral charge is obtained in this case, the form function being

$$C = \frac{C_b}{L_b} L \tag{A1.7}$$

and the web is given by equation (A1.5)

$$L_b = \frac{h}{2} \ .$$

2. Single-Perf Grains

A "single-perf" grain is a cylindrical grain pierced by a cylindrical perforation through the center of the grain parallel to the length of the grain.

Equating the charge weight to the number of grains, N, times, the density of the propellant times the volume of one grain yields

$$C_b = N \rho \frac{\pi}{4} (D^2 - d^2) h \tag{A2.1}$$

where D, d, and h are the initial values of the diameter of the grain, the diameter of the perforation, and the length of the grain, respectively. After burning through a distance, L, the charge left unburned is

$$C_b - C = N \rho \frac{\pi}{4} \left[(D - 2L)^2 - (d + 2L)^2 \right] (h - 2L) \ . \tag{A2.2}$$

Combining the two equations we have

$$C = \frac{4C_b}{h(D - d)} \left[\left(h + \frac{D - d}{2}\right) L - 2L^2 \right] \ . \tag{A2.3}$$

The grain is regressive, only slightly so if $h >> D$. Since the form function, equation (A2.3), is quadratic the value of λ_1 is

$$\lambda_1 = \frac{4C_b}{h(D - d)} \left[h + \frac{D - d}{2} \right]. \tag{A2.4}$$

The web is

$$\left. \begin{array}{l} L_b = \dfrac{D - d}{4} \\ \text{or} \\ L_b = \dfrac{h}{2} \end{array} \right\} \tag{A2.5}$$

whichever is the smaller.

If the ends of the grains are inhibited a neutral burning charge is obtained with

$$C = \frac{C_b}{L_b} L \tag{A2.6}$$

and

$$L_b = \frac{D - d}{4}. \tag{A2.7}$$

If all the surfaces except the perforation are inhibited a progressive charge is obtained with the form function

$$C = \frac{4C_b}{D^2 - d^2} (dL + L^2) \tag{A2.8}$$

with λ_1 being given by

$$\lambda_1 = \frac{4C_b d}{(D^2 - d^2)} \tag{A2.9}$$

and web,

$$L_b = \frac{D - d}{2}. \tag{A2.10}$$

3. Seven-Perf Grains

A "seven-perf" grain consists of a fairly short cylinder, pierced by seven equally spaced cylindrical perforations parallel to the length of the cylinder. It is generally specified that the diameter, d, of each perforation is approximately one-tenth the diameter, D, of the grain. The appearance of the end section is shown in Figure 13a.

The initial distance between any two perforations and between any of the outer six perforations and the curved surface of the cylinder is $\frac{D - 3d}{4}$. It will be noted that when a distance $\frac{D - 3d}{8}$ has been burned through there are twelve curvilinear, triangular prisms called "slivers" remaining unburned. (See Figure 13b.) Strictly speaking, then, a seven-perf grain has two form functions-one valid for $0 \leq L \leq \frac{D - 3d}{8}$, the other valid during burning of the slivers. In practice, however, since the grain is about 85 per cent consumed at the moment of slivering and since the latter form function is unduly complicated, it is generally assumed that the initial form function is applicable during the entire burning regime.

Equating the total charge weight, C_b, to the number of grains, N, times the density of the propellant, ρ, times the volume of one grain gives

$$C_b = N \rho \frac{\pi}{4} (D^2 - 7d^2) h \tag{A3.1}$$

where, as before, h is the length of the grain. After burning through a distance, L, the charge left unburned is

$$C_b - C = N \rho \frac{\pi}{4} \left[(D - 2L)^2 - 7(d + 2L)^2 \right] (h - 2L) . \tag{A3.2}$$

Combining the above equations the form function becomes

$$C = \frac{4C_b}{h(D^2 - 7d^2)} \left\{ [h(D + 7d) + \frac{1}{2}(D^2 - 7d^2)]L + [6h - 2(D + 7d)]L^2 - 12L^3 \right\} . \tag{A3.3}$$

The charge is progressive. The value of λ_1 for use in a quadratic form function is

$$\lambda_1 = \frac{C_b}{L_b} - \rho \frac{(S_b - S_0)}{2} = 4C_b \left[\frac{D + 7d}{D^2 - 7d^2} + \frac{1}{2h} + \frac{6L_b^2}{h(D^2 - 7d^2)} \right] . \tag{A3.4}$$

To be consistent, it becomes necessary to define the web, L_b, as the real root of equation (A3.3) when $C = C_b$. In the general case, for which $h \geq 2L_b$, we then have

$$L_b = \frac{D - \sqrt{7}d}{2(1 + \sqrt{7})} = \frac{D - 2.6458d}{7.2916} \tag{A3.5}$$

otherwise

$$L_b = \frac{h}{2} . \tag{A3.6}$$

APPENDIX B

CORRECTION FACTOR FOR RESISTANCE DUE TO SPRING

Occasionally, as in some "stores separation devices", the resistance of a spring must be overcome in addition to the other resisting forces acting on the accelerated mass. In most cases, this resistance is quite small and can be accounted for by a slight change in the potential energy correction factors derived in Section VI.

Let K be the known spring constant. The equation of motion is

$$pA = W\left(\frac{d^2x}{dt^2} + g \sin\theta\right) + Kx \tag{B.1}$$

and the energy balance equation is

$$C_v C(T_v - T) = \left(\frac{1 + K_1 + K_2}{1 + K_1}\right)\frac{W}{2}\left(v^2 + 2gx \sin\theta\right) + \frac{Kx^2}{2} . \tag{B.2}$$

If, now, the term $W g \sin\theta + Kx$ is taken as being proportional to p, we obtain the same form for the correction factors for p_{max} and v_m as was obtained in Section VI, namely

$$p^*_{max} = \left(1 + \frac{K_3}{3 - 2n}\right) p_{max} \tag{B.3}$$

and

$$v^*_m = \left(1 - \frac{1 - n}{3 - 2n} K_3\right) v_m . \tag{B.4}$$

The value of K_3 is slightly changed, however, and is obtained as follows.

From the assumption of proportionality we have

$$\int_0^x (W g \sin\theta + Kx)\,dx = \int_0^x \frac{K_3 A p}{1 + K_3}\,dx . \tag{B.5}$$

Substituting $p = \frac{(1 + K_3)}{A} W \frac{d^2x}{dt^2}$ into the foregoing equation and integrating we obtain

$$W gx \sin\theta + \frac{Kx^2}{2} = \frac{K_3 W v^2}{2} . \tag{B.6}$$

The equation must be satisfied for $v = v_m^*$ and $x = x_m$; hence, we have as the value of K_3

$$K_3 \approx \frac{W g x_m \sin\theta + \frac{1}{2} K x_m^2}{\frac{1}{2} W v_m^2} \quad . \tag{B.7}$$

APPENDIX C

APPROXIMATION OF THE ONSET RATE

The onset rate is the time rate of change of acceleration of the accelerated mass. An approximate mathematical expression for the maximum onset rate is given by

$$\dot{a}_{max} \approx k \frac{A^2}{g_c} \sqrt{\frac{12\, p^3_{max}}{U_i\, W^3}}\ \omega(\bar{\gamma}')$$

where,

$\dot{a}_{max}$ is the maximum rate of change of acceleration in g's per second, (onset rate); k is an empirical correction factor taken to be $\frac{4}{3}$.

$$\bar{\gamma}' \approx \frac{\bar{\gamma}}{1 + \frac{\lambda_2}{\lambda_1^2}\left(\frac{U_i\, p_{max}}{3F}\right)},$$

and

$$\omega(\bar{\gamma}') = \begin{cases} \bar{\gamma}'^{\frac{1}{2}} \left(\frac{2\bar{\gamma}'}{\bar{\gamma}' + 1}\right)^{\frac{3}{2}\left(\frac{\bar{\gamma}'+1}{\bar{\gamma}'-1}\right)} \left[\frac{\bar{\gamma}' + 1}{2\bar{\gamma}'} + \frac{\bar{\gamma}' - 1}{2\bar{\gamma}'}\sqrt{\frac{\bar{\gamma}' + 1}{3\bar{\gamma}' + 1}}\right]^{\frac{\bar{\gamma}'+3}{\bar{\gamma}'-1}} \left[\sqrt{\frac{\bar{\gamma}' + 1}{3\bar{\gamma}' + 1}} - \frac{\bar{\gamma}' + 1}{3\bar{\gamma}' + 1}\right], & \bar{\gamma}' \neq 1 \\[2ex] \frac{1}{2}\left(\sqrt{2} - 1\right) e^{\sqrt{2} - \frac{1}{2}}, & \bar{\gamma}' = 1 \end{cases}$$

(Values of $\omega(\bar{\gamma}')$ are plotted versus $\bar{\gamma}'$ in Figure 14.)

The approximation was obtained by substituting into the identity relationship

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} + v\left(\frac{dv}{dx}\right)^2 \tag{C.1}$$

the dimensionless variable,

$$V' = (A/\alpha BF\lambda_1)\, v, \tag{C.2}$$

and

$$X = (12A/U_i)\, x\ . \tag{C.3}$$

The distance variable X is related to the velocity variable V', by the expression

$$X = (1 - \mu V')^{-\frac{1}{\mu}} - 1 \tag{C.4}$$

where

$$\mu = \frac{\bar{\gamma}' - 1}{2} \text{ , so that equation (C.1)} \tag{C.5}$$

may be transformed into an equation relating the time rate of change of acceleration, $\dot{a}$, to the variable V'. Thus,

$$\dot{a} = \frac{(\alpha BF\lambda_1)^3}{A(U_i/12)^2}\left[V'(1-\mu V')^{2+\frac{2}{\mu}} - (1+\mu)V'^2(1-\mu V')^{1+\frac{2}{\mu}}\right] . \tag{C.6}$$

Differentiating the above equation with respect to V', equating the resulting expression to zero, and solving for V' leads to values of V' for which $\dot{a}$ is a singular point. The value of V' for which $\dot{a}$ is a maximum is subsequently found to be

$$V' = \frac{1}{\bar{\gamma}'}\left[1 - \sqrt{\frac{\bar{\gamma}' + 1}{3\bar{\gamma}' + 1}}\right]. \tag{C.7}$$

This value of V' leads to the expression for maximum $\dot{a}$:

$$\dot{a}_{max} = \frac{144\,(\alpha BF\lambda_1)^3}{AU_i^2\, g_c}\,\psi(\bar{\gamma}') \quad g\text{'s / sec.} \tag{C.8}$$

where $\psi(\bar{\gamma}')$ is a function of $\bar{\gamma}'$ only.

Assuming that p_{max} is a mathematical maximum we may substitute the expression

$$P'_{\mu} = \left(\frac{1}{\bar{\gamma}'}\right)\left(\frac{\bar{\gamma}' + 1}{2\bar{\gamma}'}\right)^{\frac{\bar{\gamma}'+1}{\bar{\gamma}'-1}} = \frac{U_i}{12W}\left(\frac{A}{\alpha BF\lambda_1}\right)^2 p_{max} \tag{C.9}$$

into equation (C.8) to yield

$$\dot{a}_{max} = \frac{A^2}{g_c}\sqrt{\frac{12\,p^3_{max}}{U_i W^3}}\;\frac{\psi(\bar{\gamma}')}{P'^{\,3/2}_{\mu}} . \tag{C.10}$$

Denoting $\psi(\bar{\gamma}')/P'^{\,3/2}_{\mu}$ by $\omega(\bar{\gamma}')$ and adding the as yet unspecified correction factor k then leads to the approximation given above.

The approximation for $\bar{\gamma}'$ was obtained in the following manner. $\bar{\gamma}'$ is related to $\bar{\gamma}$ by the expression

$$\bar{\gamma}' = \bar{\gamma} - 2\left(\frac{\alpha B}{A}\right)^2 F W \lambda_2 \quad . \tag{C.11}$$

Combining the above equation with equation (C.9) yields

$$\bar{\gamma}' = \bar{\gamma} - \frac{U_i}{6} \frac{\lambda_2 p_{max}}{\lambda_1^2 F} \bar{\gamma}' \left(\frac{2\bar{\gamma}'}{\bar{\gamma}' + 1}\right)^{\frac{\bar{\gamma}'+1}{\bar{\gamma}'-1}} \quad . \tag{C.12}$$

In practice $\bar{\gamma}'$ never differs from $\bar{\gamma}$ by more than a few percentage points so that we may approximate the expression on the right by

$$\left(\frac{2\bar{\gamma}'}{\bar{\gamma}' + 1}\right)^{\frac{\bar{\gamma}'+1}{\bar{\gamma}'-1}} = \left(1 + \frac{\bar{\gamma}' - 1}{\bar{\gamma}' + 1}\right)^{\frac{\bar{\gamma}'+1}{\bar{\gamma}'-1}} \approx 2 \; . \tag{C.13}$$

This then leads to the $\bar{\gamma}'$ approximation

$$\bar{\gamma}' \approx \frac{\bar{\gamma}}{1 + \frac{\lambda_2}{\lambda_1^2}\left(\frac{U_i \, p_{max}}{3F}\right)} \quad . \tag{C.14}$$

A comparison of values of $\dot{a}_{max}$ calculated from the above expression with those obtained experimentally indicates that the value $k = \frac{4}{3}$ for both the Mk 327 Test Set and the Dahlgren ballistic pendulum gives generally good agreement. The results of the comparison are tabulated below.

	Experimental $\dot{a}_{max}$	Calculated $\dot{a}_{max}$
Mk 327	260.1 g's / sec.	254.9 g's / sec.
Test	284.4 g's / sec.	313.3 g's / sec.
Set	222.7 g's / sec.	241.4 g's / sec.
Dahlgren	19.4 g's / sec.	18.9 g's / sec.
Ballistic	13.0 g's / sec.	12.8 g's / sec.
Pendulum	16.5 g's / sec.	16.0 g's / sec.

A "better" value of k might be obtainable from more extensive empirical data.

APPENDIX D

APPLICATION OF THE LARGER ROOT OF EQUATION 8.14

For one reason or another, the ratio of barrel volume, U_b, to chamber volume, U_c, for most CAD's is of the order of 1. For example, the ratios for the Mk 327 Test Set and the Dahlgren ballistic pendulum are 1.24 and 1.57 respectively. For ratios of this order of magnitude the methods of Section VIII are usually sufficient to determine charge weight and web size to meet given performance requirements. Recently, however, a device was encountered with a ratio $U_b / U_c = 5$. In attempting to obtain a grain design for the device it was found that no feasible solution could be obtained for any value of E through the use of Figure 7a. An investigation was undertaken to determine the reason for the failure and the means of correcting the situation. The results of the investigation follow.

In Section VIII B it is shown that for a given value of E a feasible solution exists if and only if v_m satisfies the inequality

$$\frac{\frac{1}{2} W v_m^2}{F \rho U_c} \leq \frac{V_{m1}}{2} \left[1 - \frac{U_b}{U_c} \left(\frac{1}{X_m} \right) \right] \quad ^{1} \qquad \text{(D.1)}$$

[1] The derivation, not given in Section VIII B, is as follows:

$$V_{m1} \leq V_b$$

$$\left(\frac{A}{\alpha B F \lambda} \right)^2 v_m^2 \leq \left(\frac{A}{\alpha B} \right)^2 \frac{L_b V_{m1}}{F \lambda W}$$

$$W v_m^2 \leq F \lambda L_b V_{m1} = F C_b V_{m1} = F \rho (U_c - U_i) V_{m1}$$

$$\frac{1}{2} W v_m^2 \leq \frac{V_{m1}}{2} F \rho U_c \left(1 - \frac{U_i}{U_c} \right)$$

$$\frac{\frac{1}{2} W v_m^2}{F \rho U_c} \leq \frac{V_{m1}}{2} \left[1 - \frac{U_b}{U_c} \left(\frac{1}{X_m} \right) \right]$$

where

V_{m1} is a root of the equation

$$\frac{\Gamma}{E} V_{m1}^2 = \left(1 - \frac{\bar{\gamma}-1}{2} V_{m1}\right)^{-\frac{2}{\bar{\gamma}-1}} - 1 , \tag{D.2}$$

$$X_m = \left(1 - \frac{\bar{\gamma}-1}{2} V_{m1}\right)^{-\frac{2}{\bar{\gamma}-1}} - 1 , \tag{D.3}$$

and

$$E = \frac{\frac{1}{2} W v_m^2}{p_{max} A x_m} . \tag{D.4}$$

Recall that equation (D.2) has two real roots, the smaller of which is denoted by $\tilde{V}_{m1}$. We shall denote the larger by $\check{V}_{m1}$.

The bracketed term in relation (D.1) must be positive, hence a necessary condition for the existence of a feasible solution is

$$\frac{U_b}{U_c} < X_m . \tag{D.5}$$

Referring to Figure 7a, the value of $\tilde{V}_{m1}$ corresponding to E_{max} is found to be approximately equal to $\frac{5}{2} - \bar{\gamma}$. Substitution of this value into equation (D.3) with subsequent use of the relationships (D.5) and (D.1) then yields the result:

If

$$\frac{U_b}{U_c} > R_{max} \tag{D.6}$$

where

$$R_{max} \approx \left(\frac{4}{2\bar{\gamma}^2 - 7\bar{\gamma} + 9}\right)^{\frac{2}{\bar{\gamma}-1}} - 1 \approx 3 \tag{D.7}$$

or if

$$\frac{\frac{1}{2} W v_m^2}{F \rho U_c} > \frac{\frac{5}{2} - \bar{\gamma}}{2} \left[1 - \frac{U_b}{U_c}\left(\frac{1}{R_{max}}\right)\right] \tag{D.8}$$

the larger root, $\check{V}_{m1}$, of equation (D.2) must be used. If neither (D.6) nor (D.8) is true the smaller root, $\tilde{V}_{m1}$, should be used.

In light of the above, a slight modification in the methods of Section VIII is required. In Figure 7b we have plotted $\check{V}_{m1}$ versus E and $\bar{\gamma}$. Notice that the right-hand member of the inequality (D.1) is a monotonically <u>decreasing</u> function of E when $\check{V}_{m1}$ is used, while it is monotonically <u>increasing</u> when $\tilde{V}_{m1}$ is employed. The modifications to be made are then as follows:

In Step 2 of Parts A and B of Section VIII first determine once and for all from the relationships (D.6) and (D.8) which root should be employed — similarly in Part C except that $\frac{1}{2} W v_m^2$ in (D.8) is replaced with $p_{max} A x_m E_{max}$. The rest of Part A remains unchanged. In Step 2c of Part B proceed as directed unless $\check{V}_{m1}$ is used. In that case, if $E_1 > E_2$ select a <u>smaller</u> value of E and repeat Step 2. Similarly, in Step 2d of Part C, if $E_3 > E_2$ select a smaller value of E when using $\check{V}_{m1}$.

The above procedure was used in the case of the previously mentioned device where $U_b / U_c = 5$. A feasible solution was found using $\check{V}_{m1}$ and verified by the numerical integration of the ballistic equations.

An interesting observation may be made from the relationship (D.1). If the inequality is replaced with equality, burnt occurs at the moment of completion of the stroke, since then $V_b = V_{m1}$. For a given propellant, barrel volume, ejection velocity and maximum pressure, this fact then enables us to determine two chamber volumes and two constant-surface charge configurations to give the desired performance. Furthermore, burnout in both instances should occur at end of stroke. Consider the following example.

The propellant has the following ballistic properties:

$$B = 0.01 \quad (\text{in/sec})(\text{in}^2/\text{lb}_f)^n$$

$$n = 0.5$$

$$F = 279{,}560 \ \text{ft}\cdot\text{lb}_f/\text{lb}_m$$

$$\gamma = 1.2$$

$\rho = 0.06\ \mathrm{lb}_m/\mathrm{in}^3$

$\eta = 16.67\ \mathrm{in}^3/\mathrm{lb}_m$.

The device has dimensions:

$A = 40\ \mathrm{in}^2$

$K_1 = K_2 = 0$

$W = 3108.1$ slugs

$x_m = 8$ ft; $U_b = 12\, A\, x_m = 3840\ \mathrm{in}^3$

$\theta = 0°$.

Determine two chamber volumes and their corresponding constant-surface charge configurations to yield $P_{max} = 1354$ psi and $v_m = 15$ ft/sec. Burnt should occur at ejection.

Solution

Using equality in the relation (1) we can solve for U_c to obtain

$$U_c = \frac{W v_m^2}{F \rho \tilde{V}_{m1}} + \frac{U_b}{X_m} . \qquad \text{(D.9)}$$

Since

$$\frac{U_b}{X_m} \equiv U_i \qquad \text{(D.10)}$$

equation (D.9) implies that

$$C_b \equiv \frac{W v_m^2}{F \tilde{V}_{m1}} . \qquad \text{(D.11)}$$

From the given performance requirements we calculate

$$E = \frac{\frac{1}{2} W v_m^2}{p_{max} A\, x_m} = 0.8068 \qquad \text{(D.12)}$$

and determine $\alpha = 0.036$ from Figure 1b. Using Figure 7a we obtain $\widetilde{V}_{m1} = 0.9$ and the corresponding value $X_m = 1.55$ from Figure 2b. The required charge

weight is found to be

$$C_b = Wv_m^2 / F\breve{V}_{m1} = 2.779 \text{ lb}_m \text{ ; the initial free volume is}$$

$$\frac{U_b}{X_m} = 2477 \text{ in}^3 \quad ,$$

so that $U_c = \frac{C_b}{\rho} + U_i = 2523 \text{ in}^3$.

Burnt occurs at end of stroke, hence equation (4.1) may be used to determine the web,

$$L_b = \frac{\alpha BW}{A} v_m = 0.4196 \text{ in} .$$

In summary, then, the values

$$A: \begin{bmatrix} U_c = 2523 \text{ in}^3 \\ C_b = 2.779 \text{ lb}_m \\ L_b = 0.4196 \text{ in} \end{bmatrix}$$

should produce the desired results.

Similarly, using Figures 7b and 2b we obtain $\breve{V}_{m1} = 1.8$ and $X_m = 6.4$. Employing the same procedure as above we then have

$$B: \begin{bmatrix} U_c = 623.2 \text{ in}^3 \\ C_b = 1.390 \text{ lb}_m \\ L_b = 0.4196 \text{ in} \end{bmatrix}$$

as the second set of values.

Using numerical integration techniques the ballistic performance was calculated for the two sets of values, A and B. The results are shown in Figure 15. Notice the radical difference in the shape of the curves, though they produce nearly the same p_{max} and v_m. The distance burned through the web in case A is found to be .3978 inch, while that in case B is .3586 inch.

The above process could be repeated to determine the required chamber volume for p_{max} fixed at 1354 psi and various values of v_m. By allowing v_m to take on values between 14.8 ft/sec and 15.6 ft/sec, the complete range of E given in Figures 7a and 7b may be covered. Numerical integration techniques would produce a large family of curves, two examples of which are shown in Figure 15. Depending on the desired pressure-time characteristics an "optimum" chamber volume could then be picked out from the family of curves. For example, if maximum ejection velocity is taken to be the criterion for "optimum" the procedure yields an optimum chamber volume of about 2710 in^3 for which v_m = 16.4 ft/sec and p_{max} = 1375 psi.

The foregoing suggests the possibility that for a given device with fixed dimensions, improved performance could be obtained simply by loading the chamber with inert material and decreasing the charge weight.

APPENDIX E

REFINEMENTS TO GRAIN DESIGN TO RENDER p_{max} AND v_m WITH BURNOUT NEAR END OF STROKE

Several methods of determining constant-surface charge requirements to meet given CAD performance specifications are discussed in Section VIII. One of the shortcomings common to all these methods is the fact that quite often they lead to designs that are only fractionally consumed at the end of the piston stroke. A method of correcting the situation is given in Section IX, but the correction is not always practical, especially for those CAD's having high loading densities. Furthermore, although constant-surface grain designs are usually adequate for those devices having barrel volume to chamber volume ratios ≈ 1, they generally perform poorly in those devices with ratios $>> 1$. For these reasons we return to the problem of grain design.

Let conditions a) and b) of Section VIII be replaced by the stronger conditions

a') "Burnt" shall occur at end of stroke, i. e., $V_b' = V_{m1}'$.

b') p_{max} shall be a mathematical maximum, i. e., $V_{m1}' \geq V_M' = 1/\bar{\gamma}'$.

Under these conditions we shall attempt to determine the charge weight and grain geometry required to produce a given maximum pressure and ejection velocity.

Condition a') permits the immediate determination of the web-size from equation (10.1). Thus,

$$L_b = \frac{\alpha B W}{A} v_m . \tag{E.1}$$

Employing the definition of initial free volume together with equation (10.14) yields an equation for the charge weight,

$$C_b = \rho U_c \left(1 - \frac{U_b}{U_c X_m}\right) \tag{E.2}$$

where $U_b = 12 A x_m$, the barrel volume. Condition a') also permits the rearrangement of equation (10.15) to yield an expression for λ_1, the first form-function coefficient,

$$\lambda_1 = \left(\frac{A}{\alpha BF}\right)\frac{v_m}{V'_{m1}} = \frac{W v_m^2}{F\,L_b\,V'_{m1}} \quad . \qquad \text{(E.3)}$$

Finally, equation (10.2) may be used to determine the value of λ_2, the second coefficient,

$$\lambda_2 = \frac{\lambda - \lambda_1}{L_b} = \frac{C_b/L_b - \lambda_1}{L_b} \quad . \qquad \text{(E.4)}$$

The required charge weight and grain geometry are given by these four equations. However, before they can be evaluated we must first determine V'_{m1}, and X_m from the given firing specifications.

By condition a′, V'_{m1} is related to X_m and $\bar{\gamma}'$, as before, by the expression

$$V'_{m1} = \begin{cases} \dfrac{2}{\bar{\gamma}' - 1}\left[1 - (1 + X_m)^{-\frac{\bar{\gamma}' - 1}{2}}\right], & \bar{\gamma}' \neq 1 \\ \ln\,(1 + X_m), & \bar{\gamma}' = 1 \,. \end{cases} \qquad \text{(E.5)}$$

Using a procedure identical to that employed in Section VIII we define the piezometric efficiency by

$$E = \frac{\frac{1}{2} W v_m^2}{p_{max}\,Ax_m} \qquad \text{(E.6)}$$

and the parameter Γ' by

$$\Gamma' = \begin{cases} \dfrac{\bar{\gamma}'}{2}\left(\dfrac{2\bar{\gamma}'}{\bar{\gamma}' + 1}\right)^{\frac{\bar{\gamma}' + 1}{\bar{\gamma}' - 1}}, & \bar{\gamma}' \neq 1 \\ \dfrac{e}{2}, & \bar{\gamma}' = 1 \end{cases} \qquad [1]$$

X_m, V'_{m1}, and $\bar{\gamma}'$ are then further related by the equation

$$X_m = \frac{\Gamma'}{E}\,V'^2_{m1} \quad . \qquad \text{(E.7)}$$

[1] Note that from equations (10.25) and (10.29) we have the identity relationship $\Gamma' = 1/2\;P'_M$.

Thus far we have two equations in the three unknowns X_m, V'_{m1} and $\bar{\gamma}'$. We require one more. If we define a "volumetric-efficiency" as

$$E_1 = \frac{\frac{1}{2} W v_m^2}{F \rho U_c} \tag{E.8}$$

a rather lengthy algebraic manipulation yields the third equation, [1]

$$X_m = \frac{\dfrac{U_b}{(1 - \bar{\gamma} E_1) U_c}}{1 - \dfrac{E_1}{1 - \bar{\gamma} E_1} \left(\dfrac{2}{V'_{m1}}\right) - \bar{\gamma}'} \, . \tag{E.9}$$

[1] The derivation is as follows:

$$V'_{m1} = V'_b$$

$$V'^2_{m1} = V'_b V'_{m1}$$

$$\left(\frac{A}{\alpha B F \lambda_1}\right)^2 \quad v_m^2 = \left(\frac{A}{\alpha B}\right)^2 \quad \frac{L_b}{F \lambda_1 W} V'_{m1}$$

$$W v_m^2 = \lambda_1 V'_{m1} F L_b$$

$$= \frac{\lambda_1}{\lambda} V'_{m1} F C_b$$

$$= \frac{\lambda_1}{\lambda} V'_{m1} F \rho U_c \left(1 - \frac{U_i}{U_c}\right)$$

$$E_1 = \frac{\frac{1}{2} W v_m^2}{F \rho U_c} = \frac{\lambda_1}{\lambda} \frac{V'_{m1}}{2} \left(1 - \frac{U_b}{U_c X_m}\right) \, . \tag{a}$$

Also, from equation (10.19) we have

$$\bar{\gamma}' = \bar{\gamma} - \frac{2(\frac{\lambda}{\lambda_1} - 1)}{V'_{m1}} \, .$$

Solving for $\frac{\lambda}{\lambda_1}$ yields

$$\frac{\lambda}{\lambda_1} = \frac{V'_{m1}}{2} (\gamma - \gamma') + 1 \, . \tag{b}$$

The combination of equations (a) and (b) then leads to equation (E.9).

If the given specifications satisfy certain restrictions the three non-linear equations (E.5), (E.7), and (E.9), may be solved for the variables X_m, V'_{m1} and $\bar{\gamma}'$ by any one of several possible numerical techniques. The equations (E.1) —— (E.4) may then be evaluated to determine the grain design requirements.

The numerical solution of a system of non-linear equations, however, can be quite tedious. Moreover, there is no guarantee that equations (E.5), (E.7), and (E.9) possess a real solution for a given value of E. Consequently, a less direct but simpler method of attack is suggested. Before describing the procedure, however, we shall have need of some further relationships between the variables.

In order for a grain design to be meaningful, the burning surface, S, must of course always be non-negative. In particular we must have

$$\left.\begin{aligned} S_0 &> 0 \quad {}^{1} \\ S_b &\geq 0 \end{aligned}\right\} \qquad \text{(E.10)}$$

where S_0 and S_b are, respectively, the initial and final burning surfaces. From the form function equation we obtain

$$S = \frac{1}{\rho}\frac{dC}{dL} = \frac{1}{\rho}\left(\lambda_1 + 2\lambda_2 L\right) \qquad \text{(E.11)}$$

so that the required inequality relationships become

$$\left.\begin{aligned} \lambda_1 &> 0 \\ \lambda_1 + 2\lambda_2 L_b &\geq 0 \,. \end{aligned}\right\} \qquad \text{(E.12)}$$

These inequalities restrict the permissible range of values of γ' and V'_{m1} .

The first inequality, $\lambda_1 > 0$, implies by definition that V' must be non-negative for all values of X. V'_{m1} must, therefore, be positive. Also, since $\bar{\gamma}' = 1/V'_M$, $\bar{\gamma}'$ must be non-negative.

[1] $S_0 = 0$ implies $\lambda_1 = 0$. The equations of Section X are not applicable in this case since V' and P' are then undefined.

The second inequality, $\lambda_1 + 2\lambda_2 L_b \geq 0$, implies that the ratio λ/λ_1 must satisfy

$$\frac{\lambda}{\lambda_1} \geq \frac{1}{2} \; ; \tag{E.13}$$

this follows from equation (E.4). Employing equation (b) in footnote 2, we then have another restriction on $\bar{\gamma}'$ and V'_{m1}, i. e.,

$$V'_{m1} \, (\bar{\gamma}' - \bar{\gamma}) \leq 1. \tag{E.14}$$

Combining the above restrictions with condition b') leads to the following conclusion: we need only consider those solutions to equations (E.5), (E.7), and (E.9) for which

$$\left.\begin{array}{ll} \bar{\gamma}' \geq 0 & \\ \dfrac{1}{\bar{\gamma}' - \bar{\gamma}} \leq \dfrac{1}{\bar{\gamma}'} \leq V'_{m1} & \text{for } \bar{\gamma}' < \bar{\gamma} \\ \dfrac{1}{\bar{\gamma}'} \leq V'_{m1} \leq \dfrac{1}{\bar{\gamma}' - \bar{\gamma}} & \text{for } \bar{\gamma}' \geq \bar{\gamma} . \end{array}\right\} \tag{E.15}$$

V'_{m1} and $\bar{\gamma}'$ are said to be in the "solution domain" if they satisfy these inequalities as well as equation (E.15).

We may now describe a suggested procedure for determining charge weight and grain geometry to yield a given performance.

Step 1.

Given the maximum pressure, $p^*_{\max}$ and ejection velocity, v^*_m, together with the other pertinent ballistic parameters calculate the values

$$K_3 = 2gx_m \sin\theta / v^{*2}_m$$

$$p_{\max} = p^*_{\max} \Big/ \left(1 + \frac{K_3}{3-2n}\right)$$

$$v_m = v^*_m \Big/ \left(1 - \frac{1-n}{3-2n} K_3\right)$$

$$W = (1 + K_1)\, w/gc$$

$$\bar{\gamma} = \left(\frac{1 + K_1 + K_2}{1 + K_1}\right)(\gamma - 1) + 1$$

$$E = \frac{\frac{1}{2} W v_m^2}{\phi_{max} A x_m}$$

$$E_1 = \frac{\frac{1}{2} W v_m^2}{F \rho U_c}$$

$$U_b = 12 A x_m$$

$$\mu_1 = \frac{U_b}{(1 - \bar{\gamma} E_1) U_c}$$

$$\mu_2 = \frac{E_1}{1 - \bar{\gamma} E_1}$$

Step 2.

a. Assume a value of X_m. (Initially, take $X_m = \mu_1$.)

b. From Figure 2b, select a value of $\bar{\gamma}'$ and V'_{m1} in the solution domain corresponding to X_m.[1]

c. Calculate

$$X_m = \frac{\mu_1}{1 - \mu_2\left(\frac{2}{V'_{m1}} - \bar{\gamma}'\right)} .$$

[1] The solution domain is determined in the following manner. In Figure 2b, connect those points for which $V' = \frac{1}{\bar{\gamma}'}$. Also connect those for which $V' = \frac{1}{\bar{\gamma}' - \bar{\gamma}}$. The solution domain lies within the region bounded by these two curves. In Figure 16, we have plotted the solution domain for $\bar{\gamma} = 1.295$. We shall use this figure in place of Figure 2b in a sample problem given later.

d. If the assumed value of X_m equals the calculated value, go on to part e; if not, repeat parts a. — c. using the calculated value of X_m, and the same value of $\bar{\gamma}'$.

e. From Figure 3, determine the value of P_N' corresponding to the value of $\bar{\gamma}'$.

f. Calculate

$$E = \frac{\frac{1}{2} V_{m1}'^{\,2}}{P_N' X_m} .$$

Step 3.

a. Repeat Step 2 for several different selected values of $\bar{\gamma}'$ and V_{m1}' so as to cover the solution domain.

b. Plot the values as obtained in Step 2, E and X_m versus V_{m1}'.

c. Select the value of X_m and V_{m1}' corresponding to the desired value of E as obtained in Step 1. (Note that there may be one, two, or no solutions.)

Step 4.

a. Determine $\alpha = f(n, \phi_{max})$ from Figure 1.

b. Calculate

$$L_b = \frac{\alpha B W}{A} \, v_m$$

$$C_b = \rho \, U_c \left(1 - \frac{U_b}{U_c X_m}\right)$$

$$\lambda_1 = \frac{W v_m^2}{F L_b V_{m1}'}$$

$$\lambda_2 = \frac{C_b / L_b - \lambda_1}{L_b} \text{ , and}$$

$$S = \frac{\lambda_1}{\rho} + \frac{2 \lambda_2 L}{\rho} \text{ , } 0 \le L \le L_b .$$

These equations determine the required charge weight, web-size, and surface versus burning-distance relationship to yield the given performance.

Sample Problem

Given a CAD and propellant with the following properties:

$A = 4.92\ in^2$

$B = 0.01919\ (in^2/lb_f)^n\ in/sec$

$F = 328,250\ ft.\ lb_f/lb_m$

$K_1 = 0.089$

$K_2 = 0.25$

$n = 0.372$

$U_c = 159.1\ in^3$

$w = 360\ lb_m$

$x_m = 13.474\ ft$

$\theta = 0°$

$\gamma = 1.24$

$\rho = 0.05661\ lb_m/in^3$

$\eta = 17.665\ in^3/lb_m$.

Determine a grain design to yield an ejection velocity of 120 ft/sec. The maximum pressure should be no greater than 1600 psi. Burnt should occur at or near ejection.

Solution

Proceeding according to the instructions in Step 1 we calculate

$K_3 = 0$

$p_{max} \leq 1600\ psi$

$v_m = 120\ ft/sec$

$W = 12.185$ slugs

$\bar{\gamma} = 1.295$

$E \geq 0.827$

$E_1 = 0.02967$

$U_b = 795.5 \text{ in}^3$

$\mu_1 = 5.200$

$\mu_2 = 0.030855$.

Continuing on to Step 2, we guess $X_m = \mu_1 = 5.200$. From Figure 16 we select the maximum values of $\bar{\gamma}'$ and V'_{m1} in the solution domain corresponding to $X_m = 5.2$. Thus, $\bar{\gamma}' = 0.45$; $V'_{m1} = 2.35$. We then calculate the value of X_m from the equation

$$X_m = \frac{5.2}{1 - 0.03086\left(\frac{2}{2.35} - 0.45\right)}$$

and obtain $X_m = 5.265$. Repeating the process, this time guessing $X_m = 5.265$, there is no perceptible change in V'_{m1}, so that the values

$V'_{m1} = 2.35$

$\bar{\gamma}' = 0.45$

$X_m = 5.265$

are taken as satisfying equations (E.5), (E.7), and (E.9) for some value of E. From Figure 2 we obtain $P'_M = 0.64$ corresponding to $\bar{\gamma}' = 0.45$ and calculate

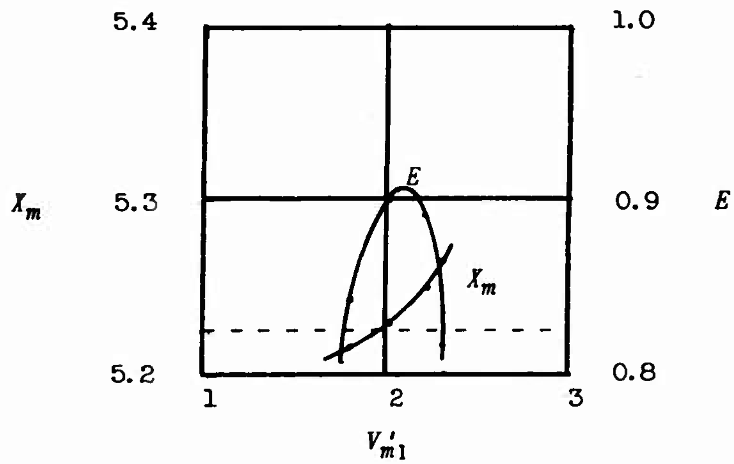
$$E = \frac{\frac{1}{2} V'^{2}_{m1}}{P'_M X_m} = 0.82.$$

In Step 3 we repeat the process for different values of $\bar{\gamma}'$ and obtain the following table.

Table 8 SOLUTIONS OF EQUATIONS (E.5), (E.7), AND (E.9) FOR VARIOUS VALUES OF E

X_m	5.2	5.265	5.2	5.249	5.2	5.231	5.2	5.218
$\bar{y}'$	0.45	0.45	0.6	0.6	0.8	0.8	1.0	1.0
V'_{m1}	2.35	2.35	2.22	2.22	2.01	2.01	1.8	1.8
X_m	5.265	5.265	5.249	5.249	5.231	5.231	5.218	5.218
P'_M		0.64		0.525		0.43		0.37
E		0.82		0.89		0.90		0.84

Since we are only interested in values of $E \geq 0.83$ we have sufficient data. The values are then plotted:



Apparently, any value of V'_{m1} between approximately 1.8 and 2.3 should meet the given specifications. Depending upon the particular choise of V'_{m1} the corresponding value of X_m may then be picked off from the graph. For our purposes we shall select

$$V'_{m1} = 2$$

$$X_m = 5.23$$

$$E = 0.9$$

and go on to Step 4.

Before we can determine α we calculate the value of p_{max} from

$$p_{max} = \frac{\frac{1}{2} W v_m^2}{E A x_m}$$

to obtain p_{max} = 1470 psi. From Figure 6a the value of α = 0.015 is obtained for n = 0.372 and p_{max} = 1470. Finally, we calculate the values

$$L_b = \frac{\alpha B W}{A} v_m = 0.08554 \text{ in}$$

$$C_b = \rho V_c \left(1 - \frac{U_b}{U_c X_m}\right) = 0.3963 \text{ lb}_m$$

$$\lambda_1 = \frac{W v_m^2}{F L_b V'_{m1}} = 3.1246 \text{ lb}_m/\text{in}$$

$$\lambda_2 = \frac{C_b/L_b - \lambda_1}{L_b} = 17.633 \text{ lb}_m/\text{in}^2$$

and $$S = \frac{\lambda_1}{\rho} + \frac{2\lambda_2 L}{\rho} = 55.20 + 623.0\, L \text{ in}^2 \quad 0 \le L \le 0.08554.$$

These values provide a preliminary design which may be modified on the basis of the results of a numerical integration of the ballistics equations and/or experimental firing results. For example, using numerical integration the above design is found to yield

p_{max} = 1420 psi, and

v_m = 112 ft/sec.

In addition, the web is found to be 80 per cent consumed at ejection. This is close to the predicted performance; however we should be able to come closer to the firing specifications by slightly modifying the design.

Let us consider the general problem of determining the required grain design modifications. We shall make the following assumptions:

(1) The form function is quadratic;

(2) Burnt occurs near the end of stroke;

(3) The loading density, $\Delta = C_b/\rho U_c$, is "small", i. e., $\Delta < 0.1$;

(4) ϕ_{max} is a mathematical maximum.

Assumptions (2), (3), and (4) are met in most CAD applications and are not felt to be serious restrictions. As for assumption (1) -- most form functions are cubics but can generally be very closely approximated by quadratic expression. At any rate, the assumptions are made for practical reasons; they lead to rather simple (and apparently accurate) correction formulas.

We shall have need of the following equations obtained from Section X:

$$P'_M = \frac{U_i}{12W}\left(\frac{A}{\alpha BF\lambda_1}\right)^2 \phi_{max} \tag{E.16}$$

$$V'_{m1} = \frac{A}{\alpha BF\lambda_1} v_m \tag{E.17}$$

$$X_m = \frac{12\, Ax_m}{U_i} \tag{E.18}$$

$$\bar{\gamma}' = \bar{\gamma} - 2\left(\frac{\alpha B}{A}\right)^2 F W \lambda_2 \tag{E.19}$$

$$\alpha = \frac{2}{n+1} \phi_{max}^{n-1} \tag{E.20}$$

$$P'_M = \frac{1}{\bar{\gamma}'}\left(\frac{\bar{\gamma}'+1}{2\bar{\gamma}'}\right)^{\frac{\bar{\gamma}'+1}{\bar{\gamma}'-1}} \tag{E.21}$$

$$V'_{m1} = \frac{2}{\bar{\gamma}'-1}\left[1 - (1 + X_m)^{-\frac{\bar{\gamma}'-1}{2}}\right] \tag{E.22}$$

$$C = \lambda_1 L + \lambda_2 L^2 . \tag{E.23}$$

Using these equations we shall derive expressions which determine the grain design modifications necessary to produce given corrections in ϕ_{max} and v_m.

Differentiating equation (E.20) we obtain

$$\frac{d\, P'_M}{P'_M} = -\,\mu\, d\,\bar{\gamma}' \tag{E.24}$$

where

$$\mu = \begin{cases} \dfrac{1}{\bar{\gamma}' - 1}\left[1 + \dfrac{2}{\bar{\gamma}' - 1}\, \ln \dfrac{\bar{\gamma}' + 1}{2\bar{\gamma}'}\right], & \bar{\gamma}' \neq 1 \\ \dfrac{3}{4}, & \bar{\gamma}' = 1 . \end{cases} \tag{E.25}$$

The assumption of small loading density permits us to treat the initial free volume, U_i, as being relatively independent of small changes in the grain design, i. e., $U_i \approx$ constant. Therefore, equation (E.22) may be differentiated to yield approximately,

$$\frac{d\,V'_{m1}}{V'_{m1}} = -\,\nu'\, d\,\bar{\gamma}' \tag{E.26}$$

where

$$\nu' = \begin{cases} \dfrac{1}{\bar{\gamma} - 1}\left[1 - \left(\dfrac{1}{V'_{m1}} - \dfrac{\bar{\gamma}' - 1}{2}\right) \ln\,(1 + X_m)\right]; & \bar{\gamma}' \neq 1 \\ \dfrac{V'_{m1}}{2}, & \bar{\gamma}' = 1 . \end{cases} \tag{E.27}$$

There are essentially three grain design parameters that determine ballistic performance: the web length, L_b, and the two form function coefficients λ_1 and λ_2. We consider the effects of holding any two of them constant and varying the third.

Case 1 - - λ_1, λ_2 constant; L_b variable

The web length, L_b, enters equations (E.16) — (E.23) only indirectly through its effect on the charge weight, C_b, and the consequent effect on the initial free volume, U_i. In the case of small loading densities, we can then say that for constant λ_1 and λ_2 a small change in L_b produces no appreciable change in performance.

Case 2 - - λ_1, L_b constant; λ_2 variable

Differentiating equations (E.16) and (E.19) we obtain

$$\frac{d\,P'_M}{P'_M} = (3-2n)\,\frac{d\,\phi_{max}}{\phi_{max}} \tag{E.28}$$

and,

$$d\,\bar{\gamma}' = (\bar{\gamma} - \bar{\gamma}')\left[2\,(1-n)\,\frac{d\,\phi_{max}}{\phi_{max}} - \frac{d\,\lambda_2}{\lambda_2}\right] . \tag{E.29}$$

Combining equations (E.24), (E.28), and (E.29) then leads to

$$\frac{d\, p_{max}}{p_{max}} = \frac{\mu\, (\bar{\gamma} - \bar{\gamma}')}{3 - 2n + 2\mu\, (\bar{\gamma} - \bar{\gamma}')\, (1 - n)}\, \frac{d\, \lambda_2}{\lambda_2}\, . \qquad \text{(E.30)}$$

Replacing differentials with finite differences the equation, (E.30), expresses the approximate fractional change in p_{max} due to a fractional change in λ_2.

Similarly, differentiating equation (E.17) and combining the result with equations (E.26), (E.24), (E.28), and (E.30), we obtain

$$\frac{d\, v_m}{v_m} = \frac{(\bar{\gamma} - \bar{\gamma}')\, [\nu'(3 - 2n) - \mu\, (1 - n)]}{3 - 2n + 2\mu\, (\bar{\gamma} - \bar{\gamma}')\, (1 - n)}\, \frac{d\, \lambda_2}{\lambda_2}\, , \qquad \text{(E.31)}$$

which relates the change in ejection velocity to a small change in λ_2.

<u>Case 3</u> - - λ_2, L_b constant; λ_1 variable

Again differentiating equations (E.16) and (E.19) we obtain in this case

$$\frac{d\, P'_M}{P'_M} = (3 - 2n)\, \frac{d\, p_{max}}{p_{max}} - 2\, \frac{d\, \lambda_1}{\lambda_1}\, , \qquad \text{(E.32)}$$

and,

$$d\, \bar{\gamma}' = 2\, (\bar{\gamma} - \bar{\gamma}')\, (1 - n)\, \frac{d\, p_{max}}{p_{max}}\, . \qquad \text{(E.33)}$$

These equations, together with equation (E.24), then lead to

$$\frac{d\, p_{max}}{p_{max}} = \frac{2}{3 - 2n + 2\mu\, (\bar{\gamma} - \bar{\gamma}')\, (1 - n)}\, \frac{d\, \lambda_1}{\lambda_1}\, . \qquad \text{(E.34)}$$

Proceeding as in Case 2, we also obtain

$$\frac{d\, v_m}{v_m} = \frac{1 - 2\, (\bar{\gamma} - \bar{\gamma}')\, (1 - n)\, (2\nu' - \mu)}{3 - 2n + 2\mu\, (\bar{\gamma} - \bar{\gamma}')\, (1 - n)}\, \frac{d\, \lambda_1}{\lambda_1}\, . \qquad \text{(E.35)}$$

The results of the three cases are summarized in the following table,

Table 9 APPROXIMATE CHANGES IN MAXIMUM PRESSURE AND EJECTION VELOCITY DUE TO SMALL CHANGES IN GRAIN DESIGN PARAMETERS

1 per cent increase in:	Percentage increase in:	
	p_{max}	v_m
L_b; (λ_1, λ_2 constant)	0	0
$\lvert\lambda_2\rvert$;* (λ_1, L_b constant)	$\frac{\mu(\bar{\gamma}-\bar{\gamma}')}{\beta'}$	$\frac{\bar{\gamma}-\bar{\gamma}'}{\beta'}\left[\nu'(3-2n)-\mu(1-n)\right]$
λ_1; (λ_2, L_b constant)	$\frac{2}{\beta'}$	$\frac{1}{\beta'}\left[1-2(\bar{\gamma}-\bar{\gamma}')(1-n)(2\nu'-\mu)\right]$
*Absolute value of λ_2		

where

$$\mu = \begin{cases} \frac{1}{\bar{\gamma}'-1}\left[1+\frac{2}{\bar{\gamma}'-1}\ \ln\frac{\bar{\gamma}'+1}{2\bar{\gamma}'}\right], & \bar{\gamma}' \neq 1 \\ \frac{3}{4}, & \bar{\gamma}' = 1 \end{cases}$$

$$\nu' = \begin{cases} \frac{1}{\bar{\gamma}'-1}\left[1-\left(\frac{1}{V'_{m1}}-\frac{\bar{\gamma}'-1}{2}\right)\ln\ (1+X_m)\right], & \bar{\gamma}' \neq 1 \\ V'_{m1}/2, & \bar{\gamma}' = 1 \end{cases}$$

$$\beta' = 3 - 2n + 2\mu\ (\bar{\gamma} - \bar{\gamma}')\ (1 - n)$$

$$\bar{\gamma}' = \bar{\gamma} - 2\left(\frac{\alpha B}{A}\right)^2 F\,W\,\lambda_2$$

$$V'_{m1} = \frac{A}{\alpha B F \lambda_1}\ v_m$$

$$X_m = 12\ A x_m / U_i\ .$$

To illustrate the use of Table 9, let us restate our sample problem.

Given a CAD and propellant with the following properties:

A = 4.92 in^2

B = 0.01919 (in^2/lb$_f$)^n in/sec

F = 328,250 ft·lb$_f$/lb$_m$

$n = 0.372$

$U_c = 159.1 \text{ in}^3$

$W = 12.185$ slugs

$x_m = 13.474$ ft

$\theta = 0$ degrees

$\bar{\gamma} = 1.295$

$\rho = 0.05661 \text{ lb}_m/\text{in}^3$

$\eta = 17.67 \text{ in}^3/\text{lb}_m$

We wish to determine a grain design to yield an ejection velocity of 120 ft/sec. The maximum pressure must not exceed 1600 psi. Burnt should occur at or near ejection.

Employing the method of this appendix, a tentative design is obtained having the values

$\lambda_1 = 3.1246 \text{ lb}_m/\text{in}$

$\lambda_2 = 17.633 \text{ lb}_m/\text{in}^2$

$L_b = 0.08554$ in

$C_b = 0.3963 \text{ lb}_m$

$S = \lambda_1/\rho + 2\,\lambda_2\,L/\rho = 55.20 + 623.0\,L \text{ in}^2$

A test firing of the charge is made and the following results are obtained:

$v_m = 111.9$ ft/sec

$p_{max} = 1418$ psi

L_m, the distance burned through the grain at ejection = 0.06733 in. What modifications should be made to yield the specified performance requirements?

Solution

We observe that v_m must be increased by about 7.2 per cent without increasing ϕ_{max} by more than 12.8 per cent. Using Table 9, we then proceed as follows.

Calculate the values:

$$\alpha = \frac{2}{n+1} \phi_{max}^{n-1} = 0.0153$$

$$\bar{\gamma}' = \bar{\gamma} - 2\left(\frac{\alpha B}{A}\right)^2 F W \lambda_2 = 0.7930$$

$$V'_{m1} = \frac{A}{\alpha B F \lambda_1} v_m = 1.829$$

$$X_m = \frac{12 \, Ax_m}{U_i} = 5.230$$

$$\mu = \frac{1}{\bar{\gamma}' - 1}\left[1 + \frac{2}{\bar{\gamma}' - 1} \ln \frac{\bar{\gamma}' + 1}{2\bar{\gamma}'}\right] = 0.8942$$

$$\nu' = \frac{1}{\bar{\gamma}' - 1}\left[1 - \left(\frac{1}{V'_{m1}} - \frac{\bar{\gamma}' - 1}{2}\right) \ln (1 + X_m)\right] = 0.9155$$

$$\beta' = 3 - 2n + 2\mu(\bar{\gamma} - \bar{\gamma}')(1 - n) = 2.820 .$$

Table 9 then has the numerical form:

Table 10 APPROXIMATE CHANGES IN ϕ_{max} AND v_m DUE TO A ONE PER CENT CHANGE IN DESIGN PARAMETERS - SAMPLE PROBLEM

1 per cent increase in:	Percentage increase in:	
	ϕ_{max}	v_m
L_b	0	0
$\lvert\lambda_2\rvert$	0.1592	0.2677
λ_1	0.7092	0.1452

In order to determine the corrected values of λ_1 and λ_2 we shall arbitrarily set as the desired value of the maximum pressure, $p_{max} = 1575$ psi. We must then select new values of λ_1 and λ_2 such that the previously attained values $p_{max} = 1418$ psi and $v_m = 111.9$ ft/sec, are increased by 11.1 per cent and 7.2 per cent respectively. Since for small changes in λ_1 and λ_2 the percentage changes in ballistic performance are approximately additive we may form the two equations:

$$0.7092\,\epsilon_1 + 0.1592\,\epsilon_2 = 11.1$$

$$0.1452\,\epsilon_1 + 0.2677\,\epsilon_2 = 7.2$$

where ϵ_1 and ϵ_2 are, respectively, the percentage changes in λ_1 and $|\lambda_2|$ required to produce the desired changes in p_{max} and v_m.

Solving for ϵ_1 and ϵ_2 we obtain

$$\epsilon_1 = 10.4 \text{ per cent (increase in } \lambda_1)$$

$$\epsilon_2 = 21.0 \text{ per cent (increase in } |\lambda_2|).$$

The corrected form function coefficients are then

$$\lambda_1 = (1.104)\ (3.1246) = 3.450 \text{ lb}_m/\text{in}$$

$$\lambda_2 = (1.210)\ (17.633) = 21.336 \text{ lb}_m/\text{in}^2.$$

Also, to reduce the amount of unburned propellant at ejection we shall decrease the web to 0.0675 in. According to Table 10, this will have no appreciable effect on performance. The new charge weight thus becomes

$$C_b = \lambda_1 L_b + \lambda_2 L_b^2 = 0.3301 \text{ lb}_m.$$

In summary, then, the recommended new design parameters are

$$\lambda_1 = 3.450 \text{ lb}_m/\text{in}$$

$$\lambda_2 = 21.336 \text{ lb}_m/\text{in}^2$$

$$L_b = 0.0675 \text{ in}$$

$$C_b = 0.3301 \text{ lb}_m$$

$$S = 60.94 + 753.8\,L \text{ in}^2$$

These values determine the required grain geometry. Substituting the values into the equations of Appendix A, leads to the selection of a single-perf grain design with an outer diameter of 0.30 in and a perforation diameter of 0.16 in, all surfaces except the perforation being inhibited.

APPENDIX F

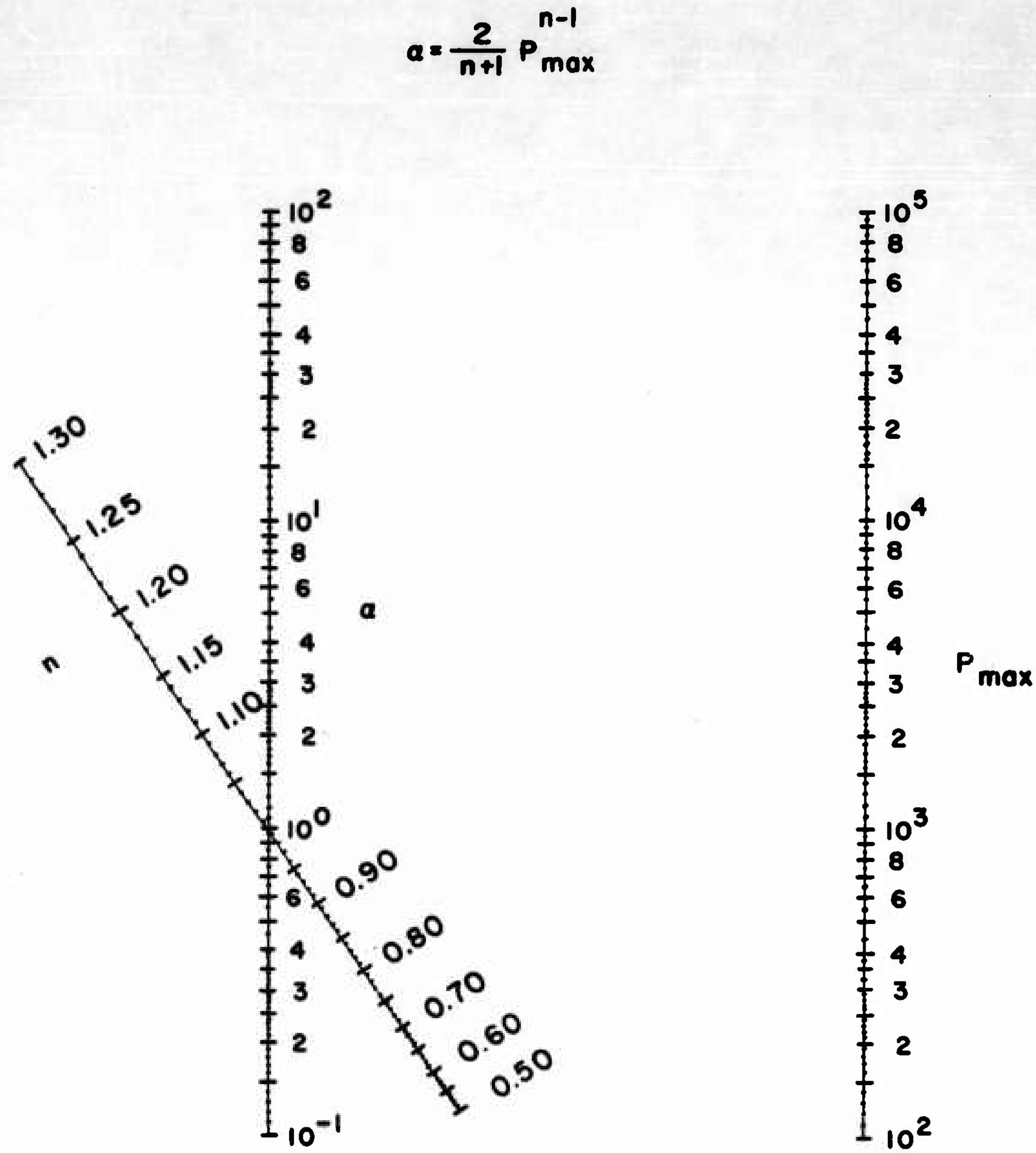


Figure 1A: a Versus n and P_{max}

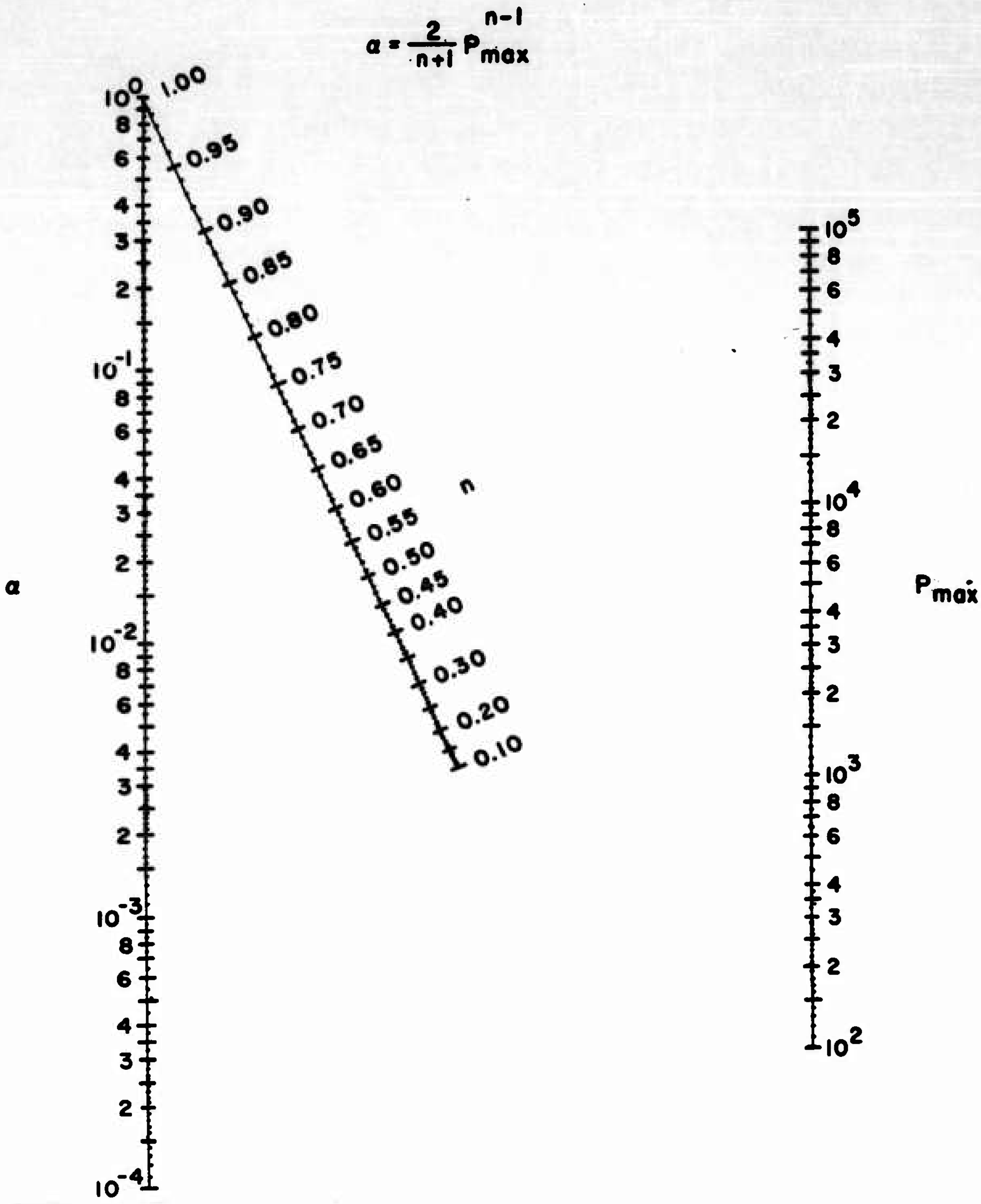


Figure 1B: α Versus n and P_{max}

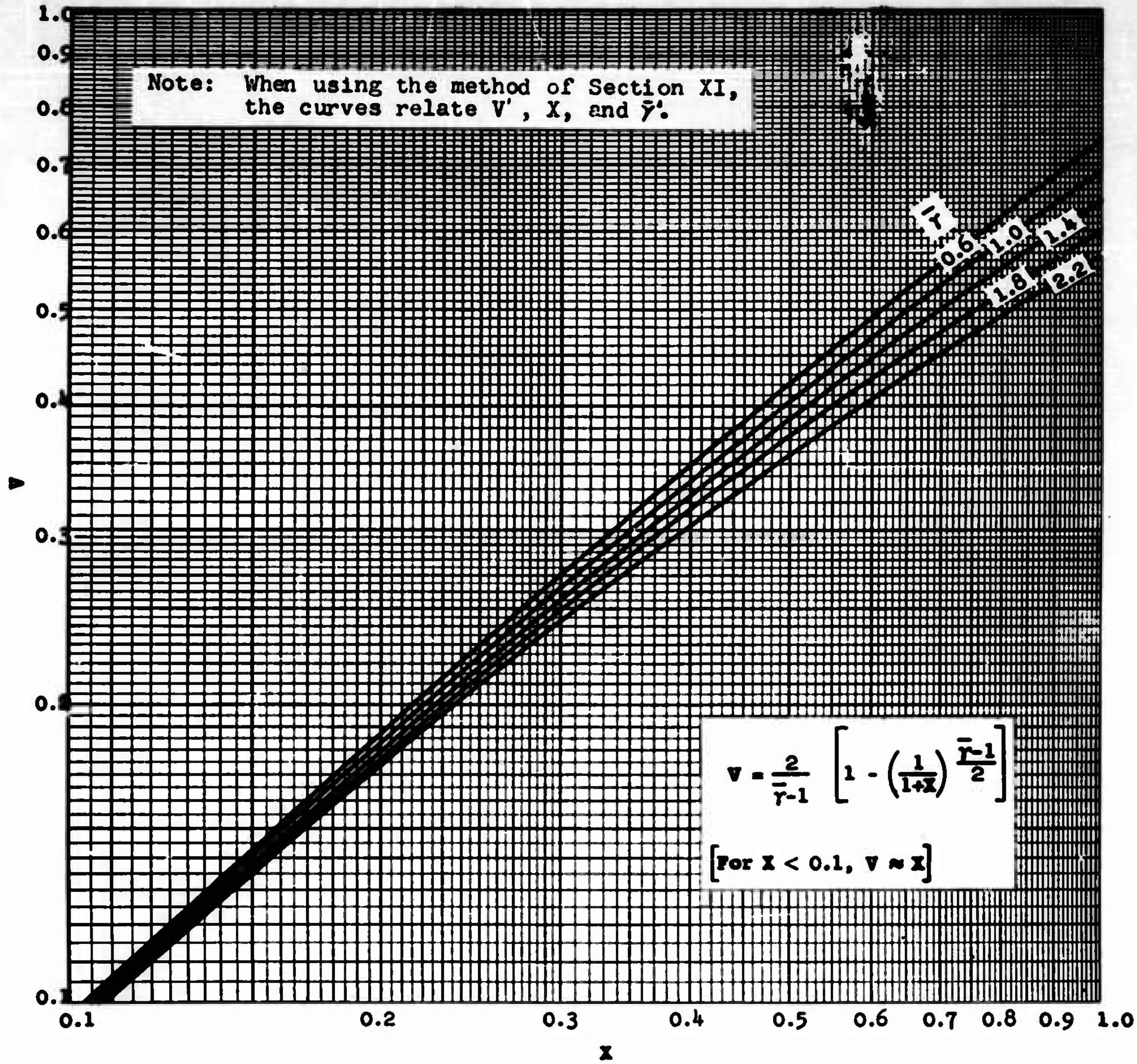


Figure 2A: V Versus X and $\bar{\gamma}$ (during burning)

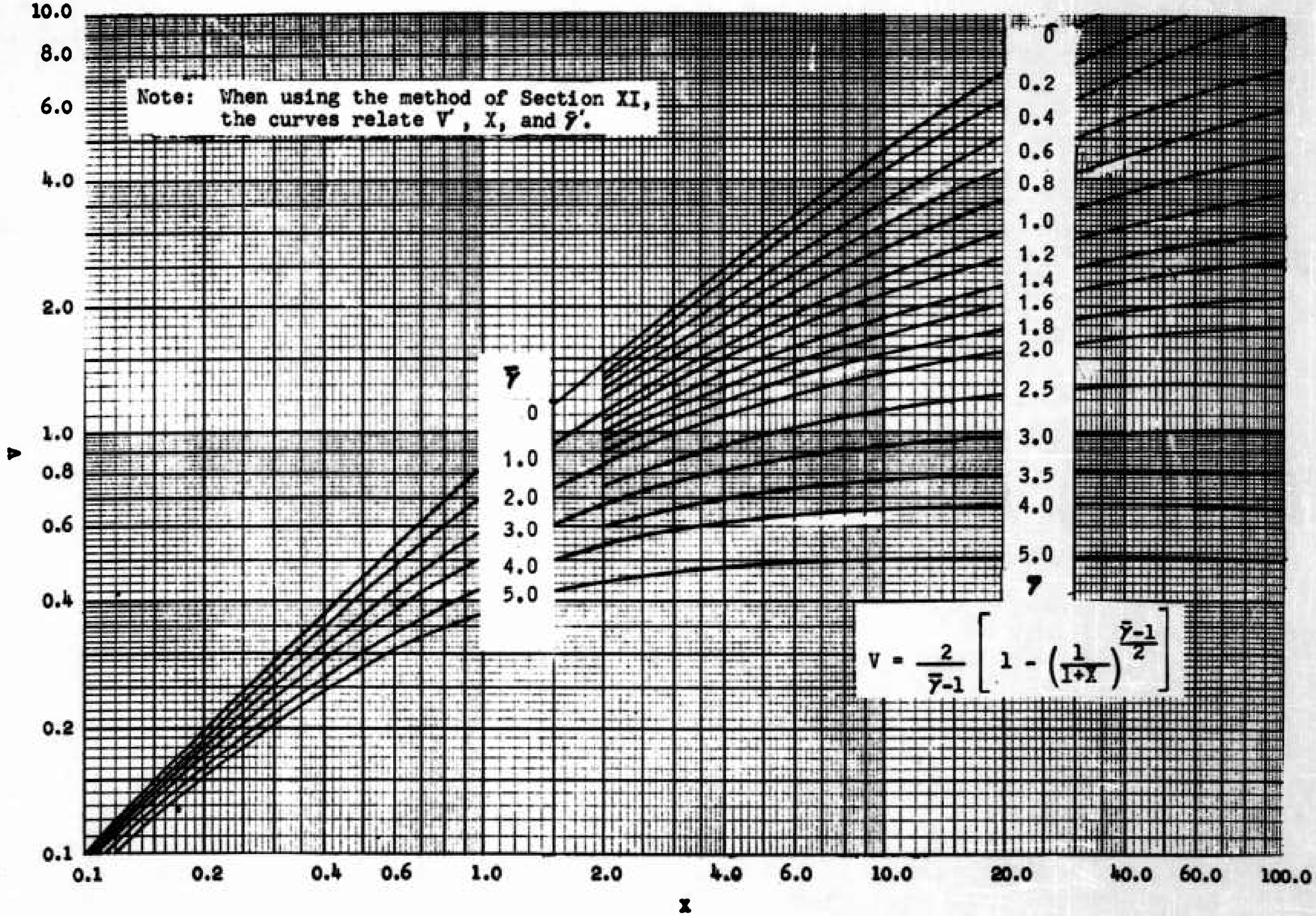


Figure 23: V Versus X and $\bar{\gamma}$ (during burning)

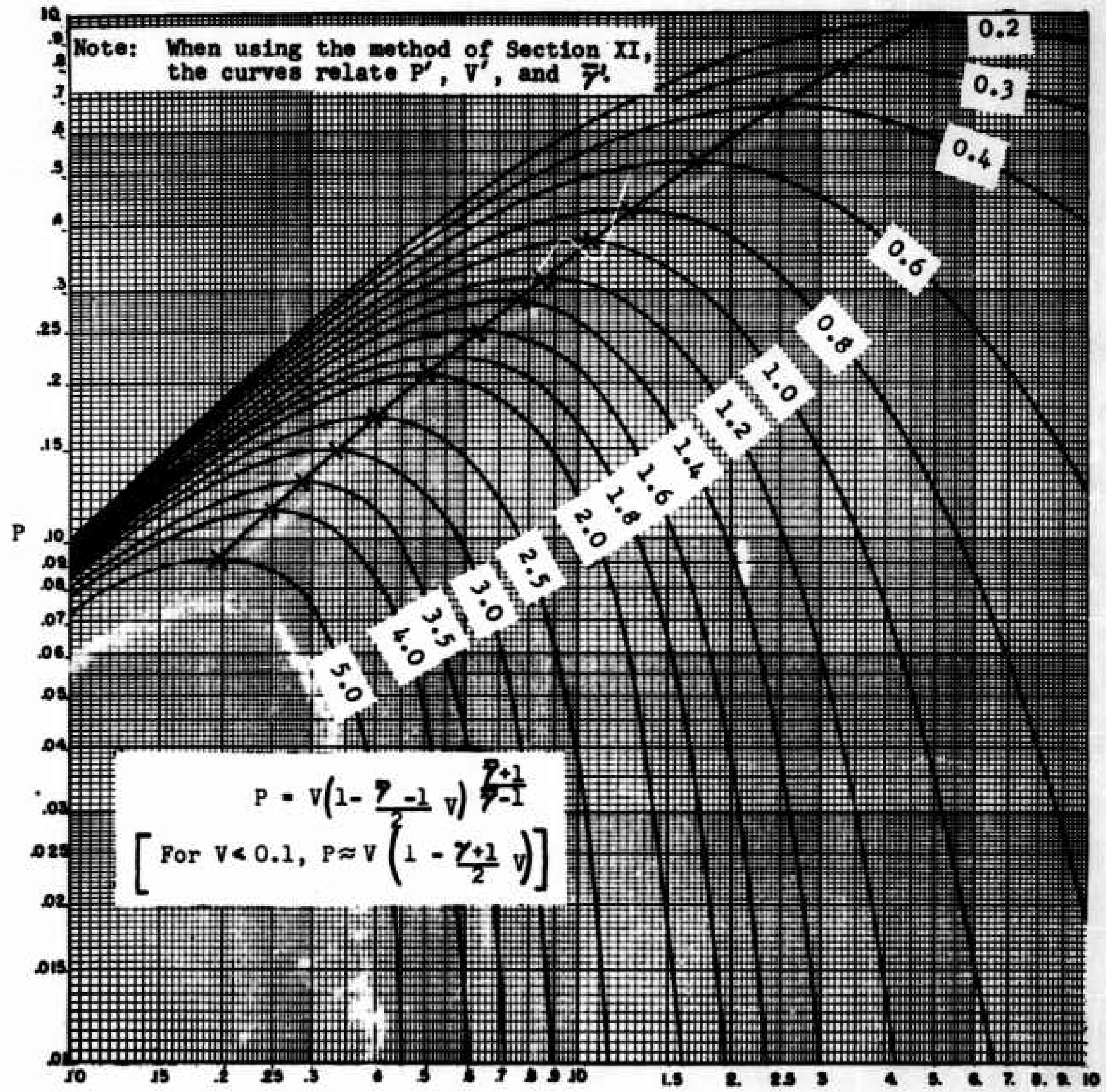


Figure 3: P Versus V and $\bar{\gamma}$ (during burning)

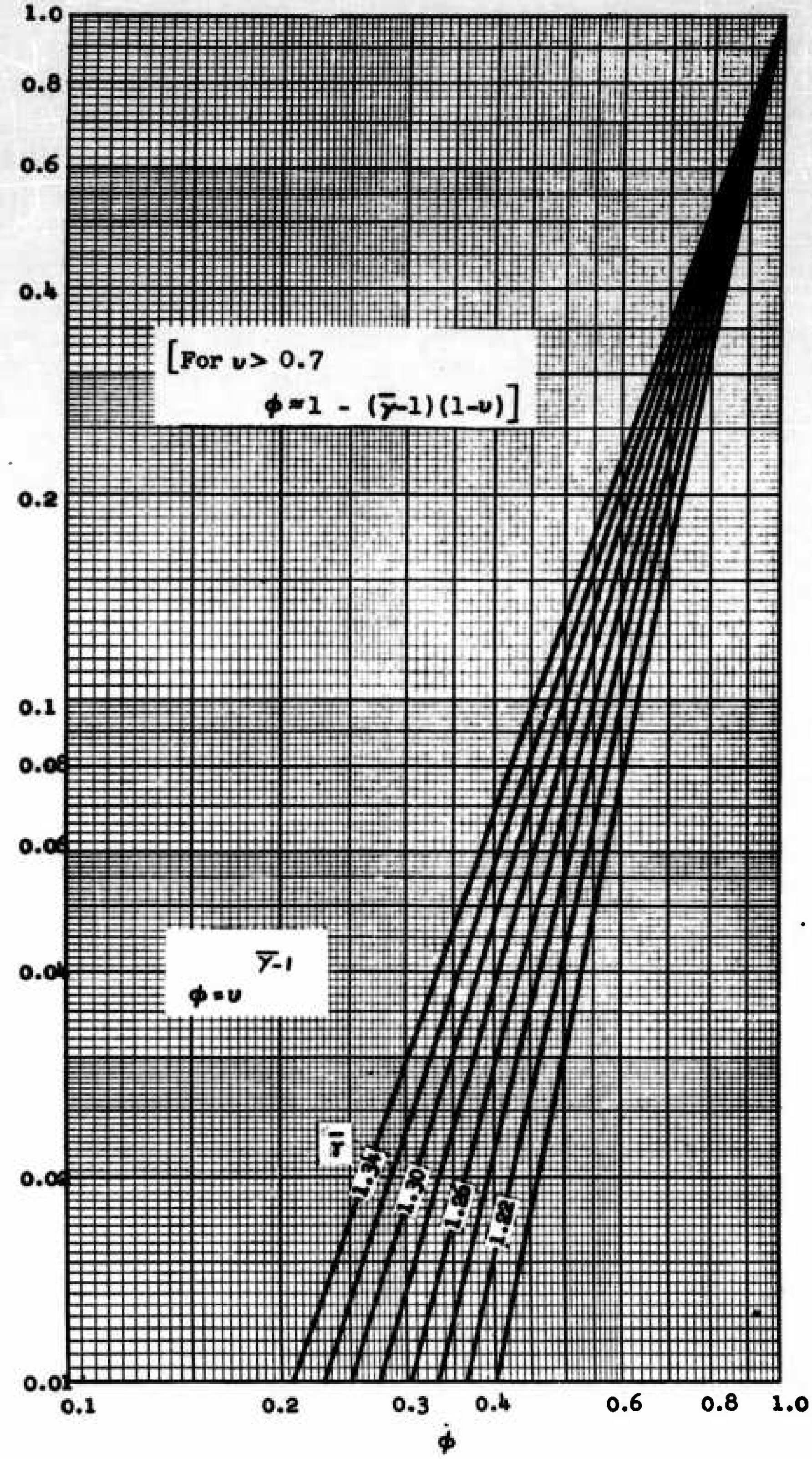


Figure 4: ϕ Versus ν and $\bar{\gamma}$

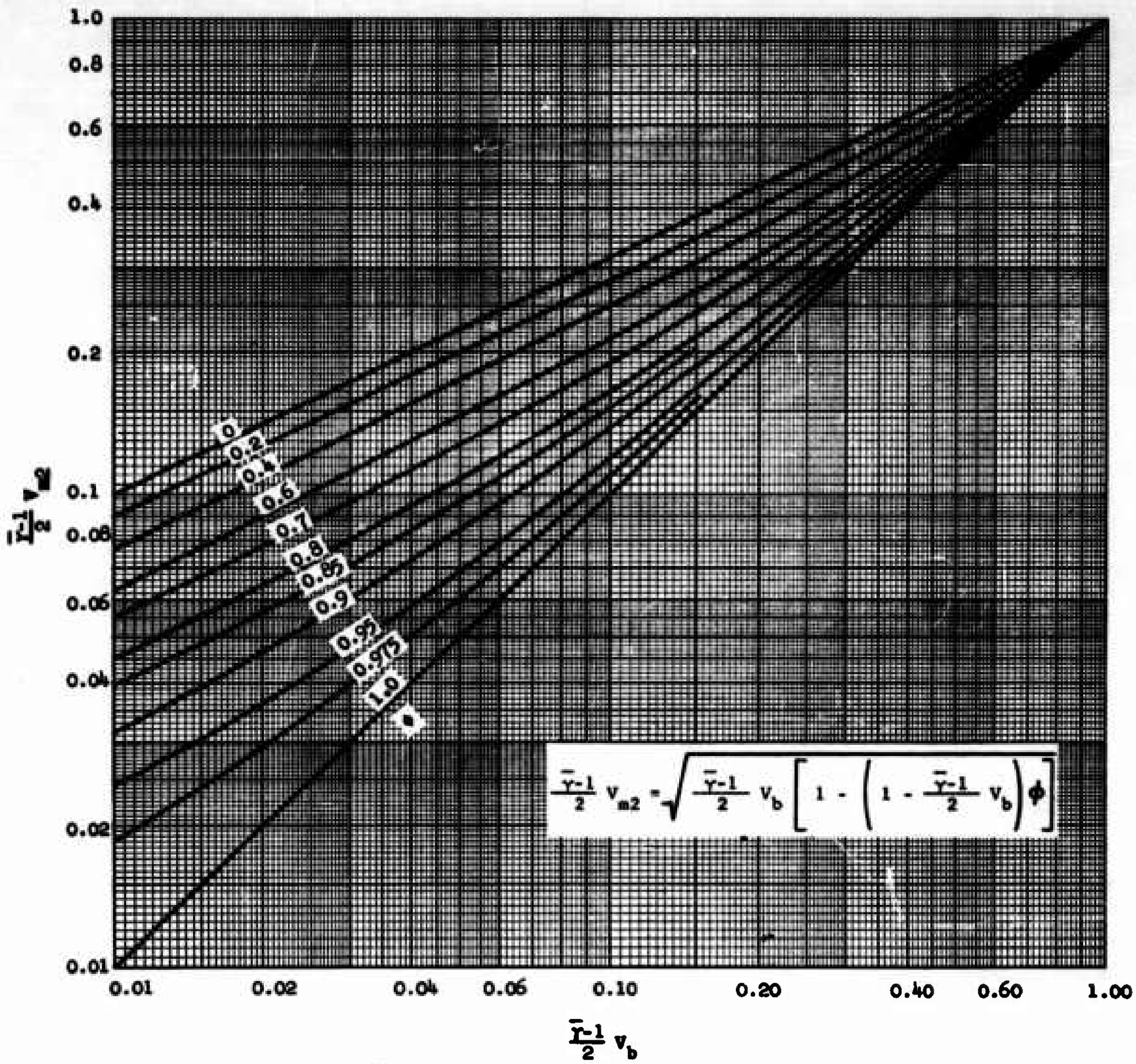


Figure 5: $\frac{\bar{\gamma}-1}{2} V_{m2}$ Versus $\frac{\bar{\gamma}-1}{2} V_b$ and ϕ

Note: When using the method of Section XI, the nomograph relates α, n, and P'_μ/q'_1.

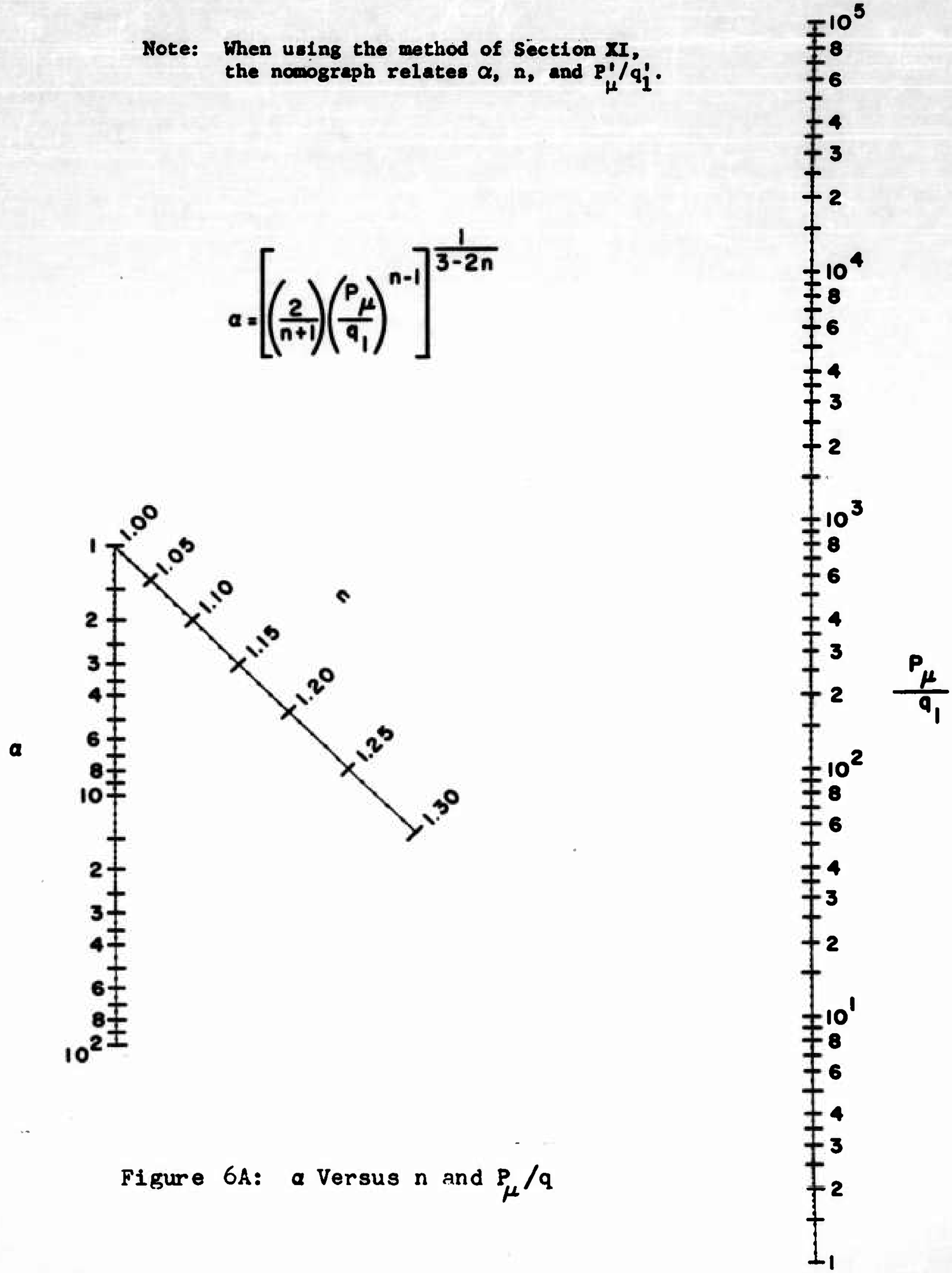


Figure 6A: α Versus n and P_μ/q

Note: When using the method of Section XI, the nomograph relates α, n, and P'_μ/q'_1.

$$\alpha = \left[\left(\frac{2}{n+1}\right)\left(\frac{P_\mu}{q_1}\right)^{n-1}\right]^{\frac{1}{3-2n}}$$

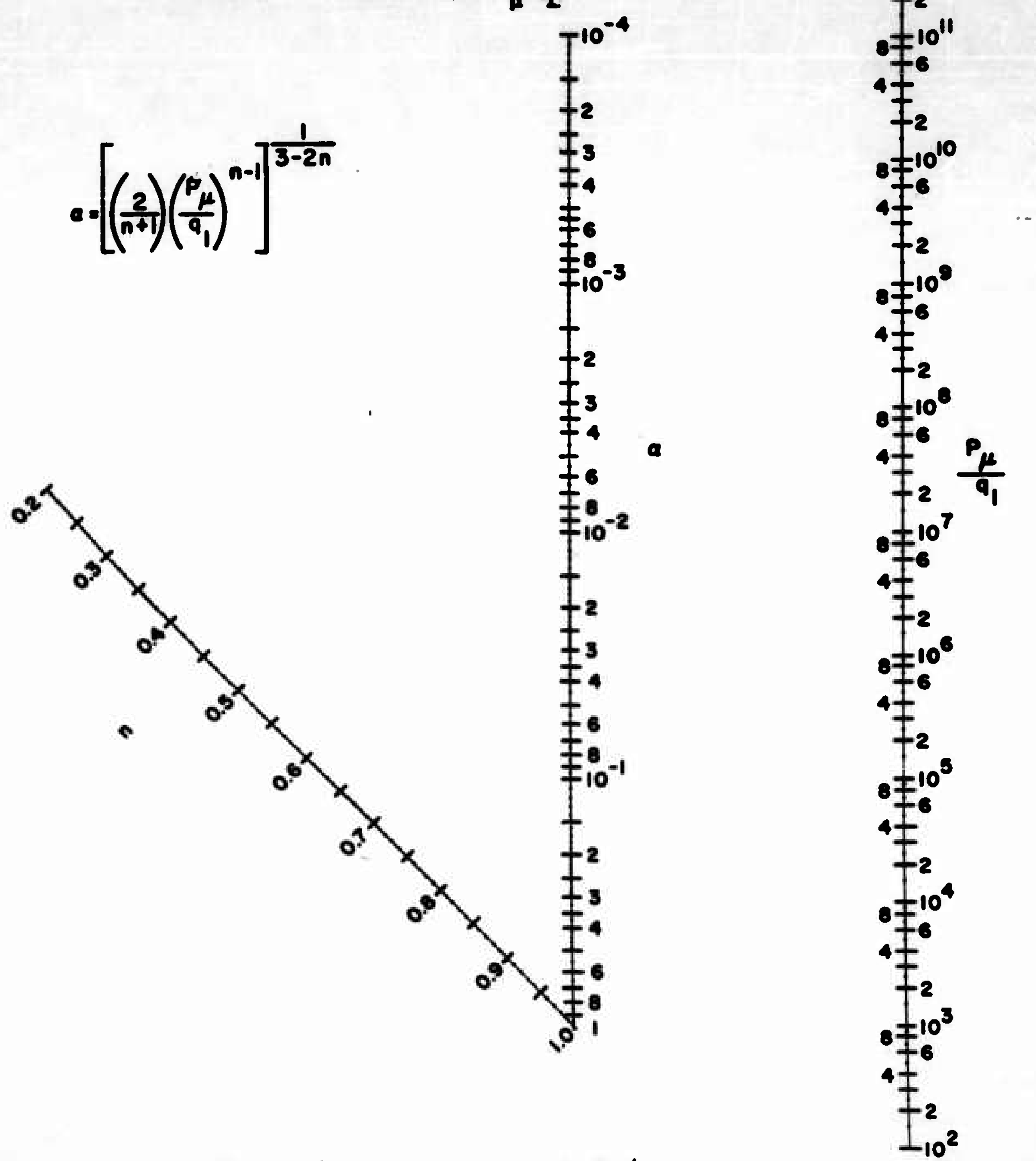


Figure 6B: α Versus n and P_μ/q

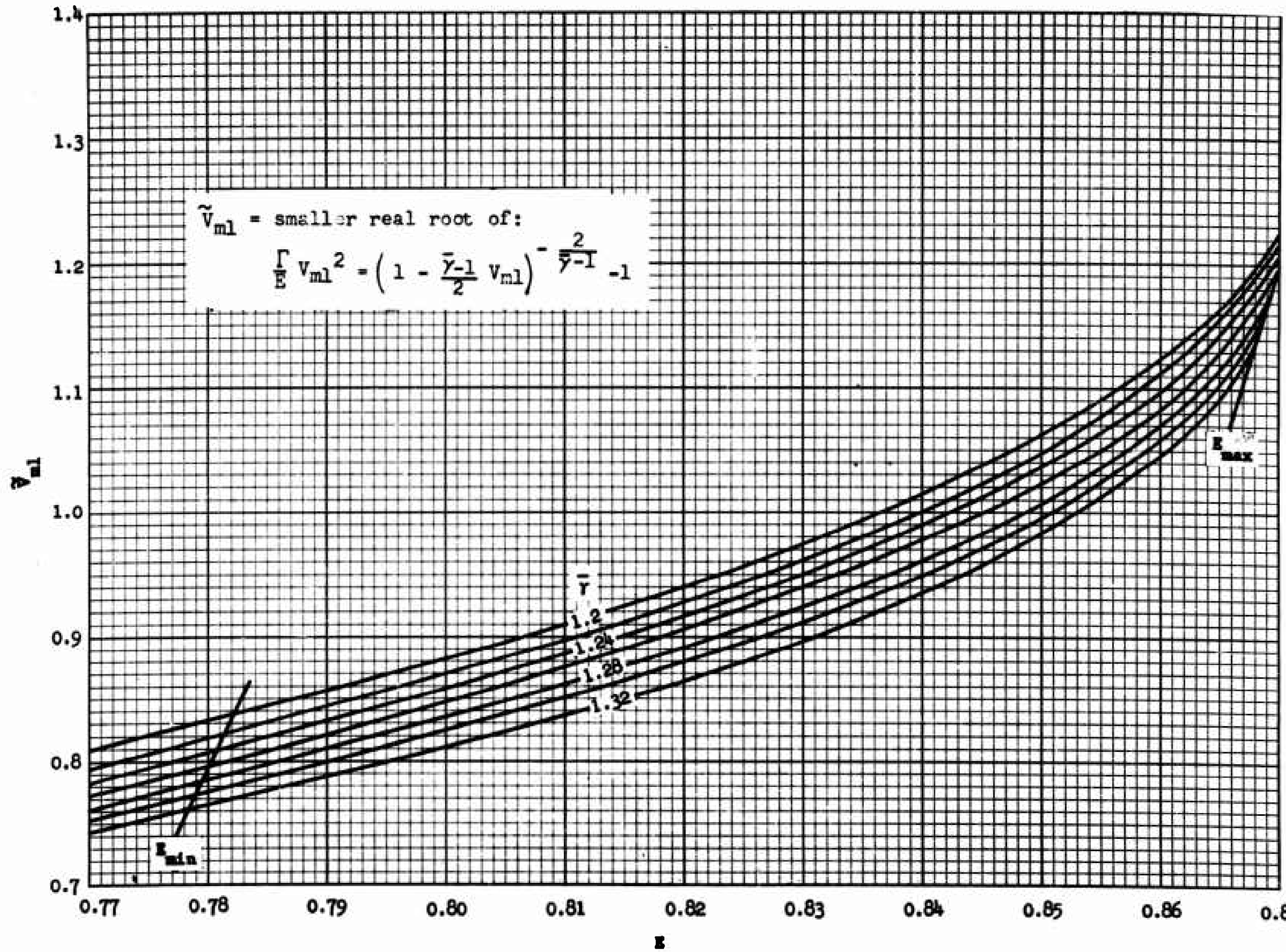


Figure 7A: $\tilde{V}_{m1}$ Versus E and $\bar{\gamma}$

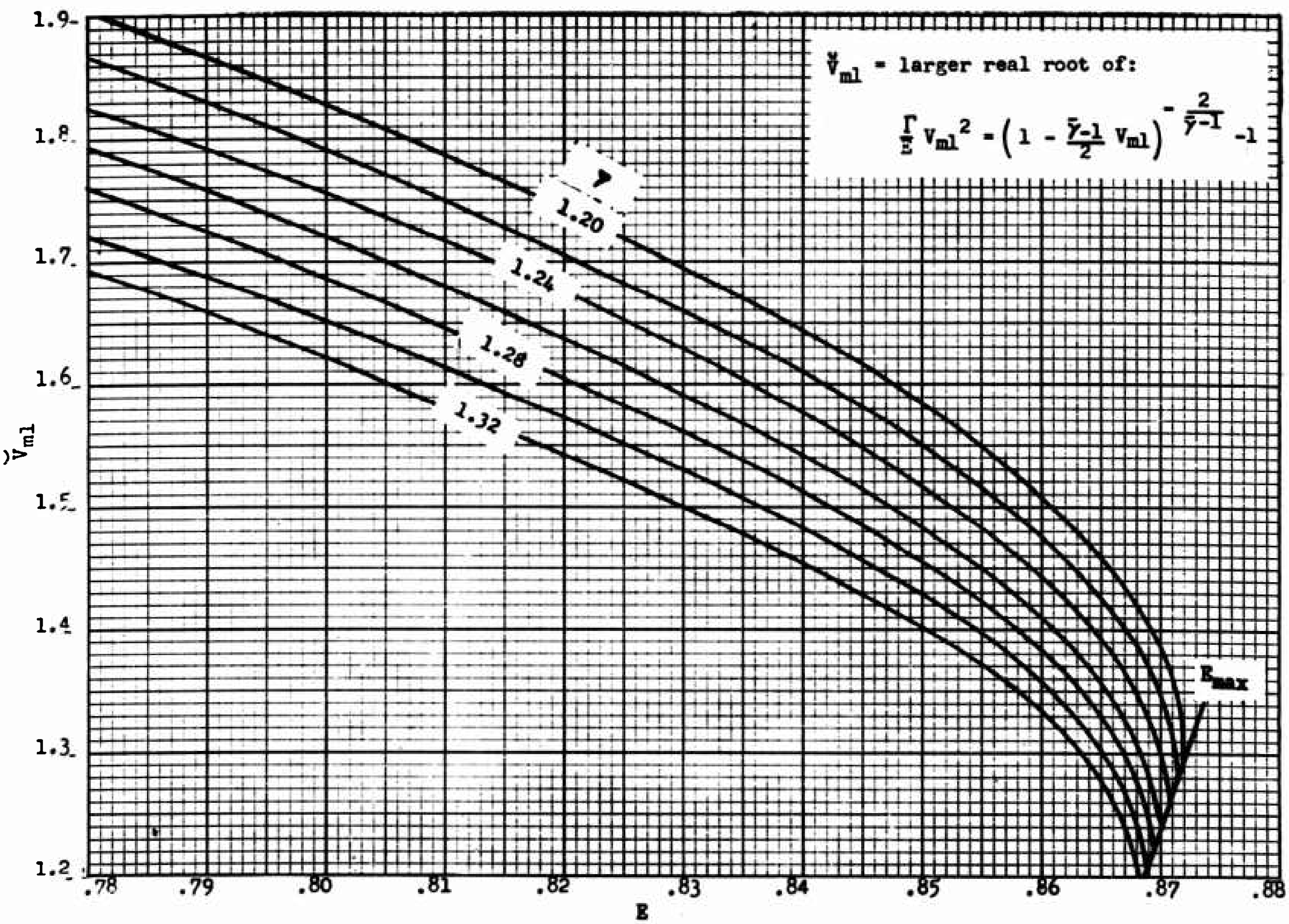


Figure 73: $\breve{V}_{ml}$ Versus E and $\bar{\gamma}$

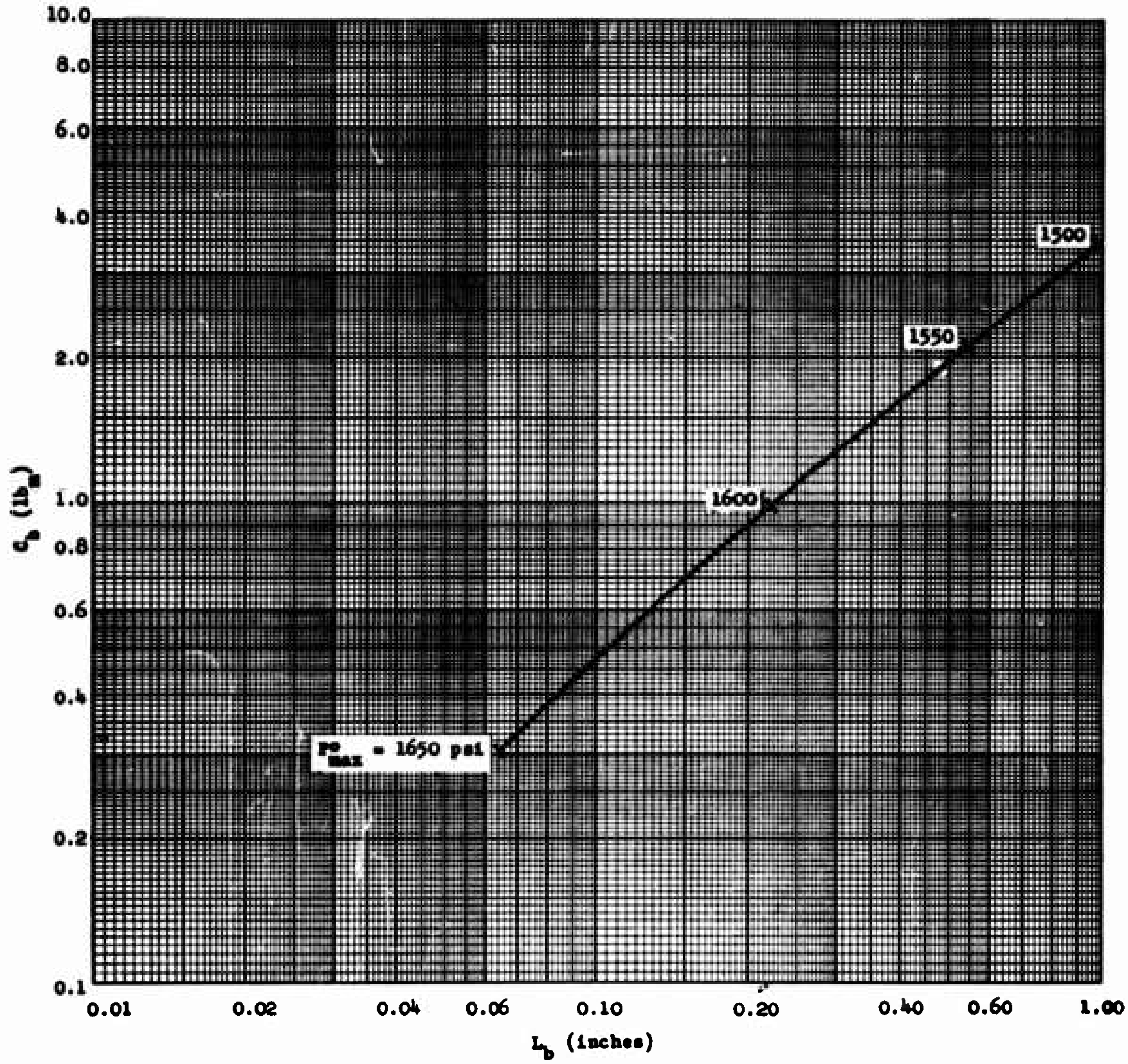


Figure 8. Values of Charge Weight, C_b, Versus Web Size, L_b, to Yield an Ejection Velocity of 60 ft/sec

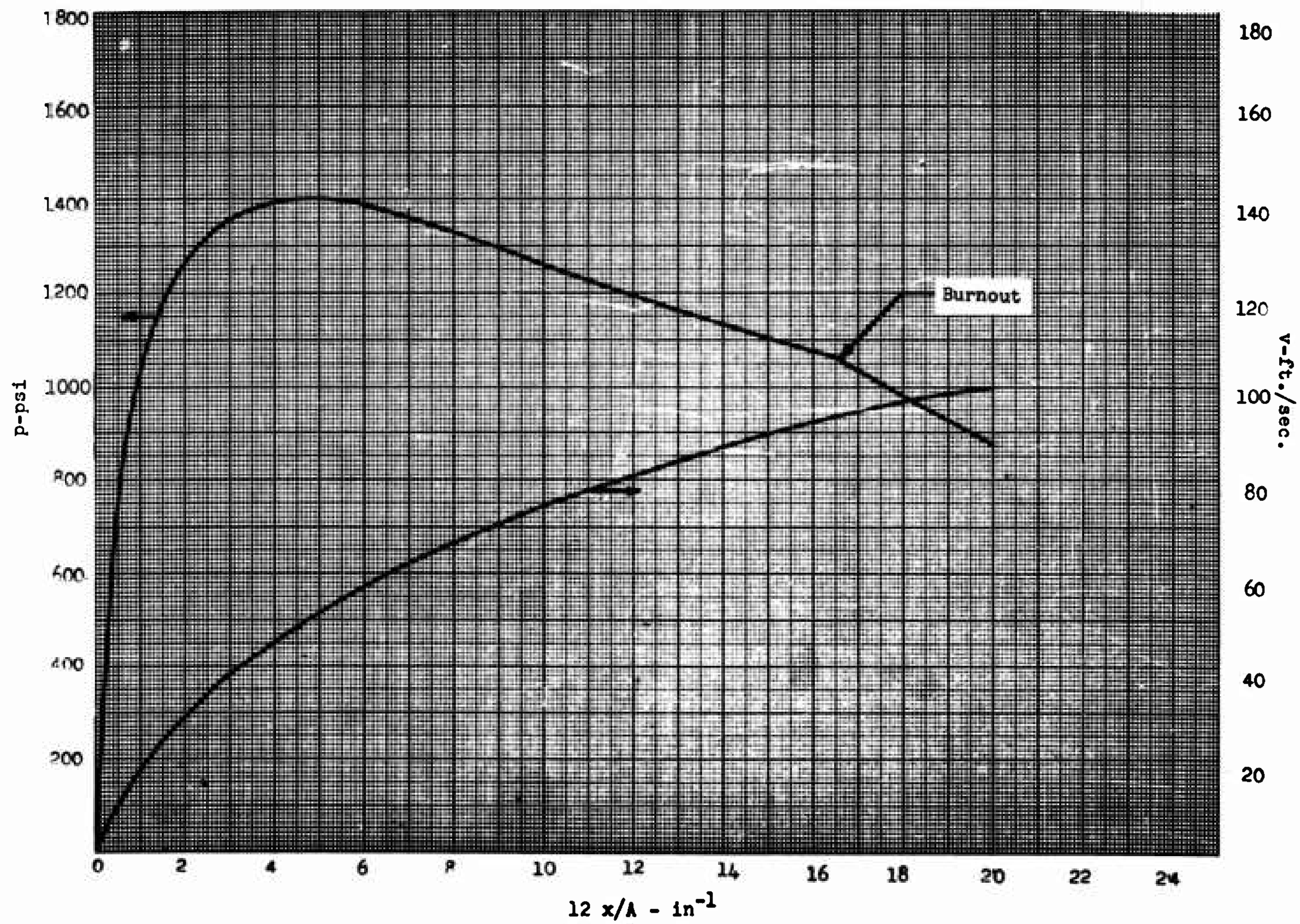


Figure 9: $\begin{Bmatrix}\text{Pressure}\\\text{Velocity}\end{Bmatrix}$ Versus the Ratio $\frac{\text{Displacement}}{\text{cross-sectional area}} = \frac{12x}{A}$ for Both Parent and Model Devices; Case I

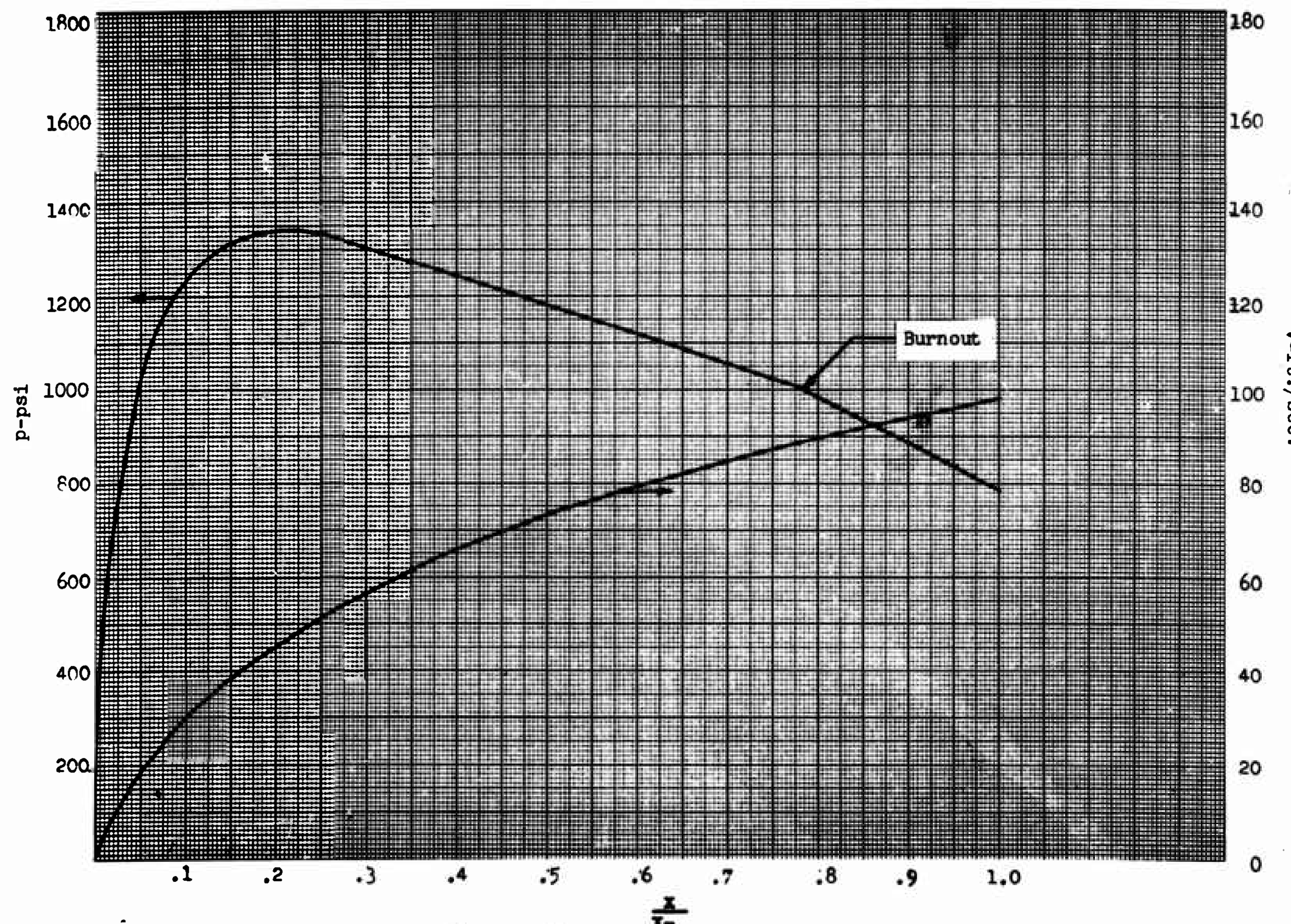


Figure 10: $\left\{\begin{matrix}\text{Pressure}\\\text{Velocity}\end{matrix}\right\}$ Versus Displacement Fraction $\frac{x}{x_m}$, for

Devices I, II, and III

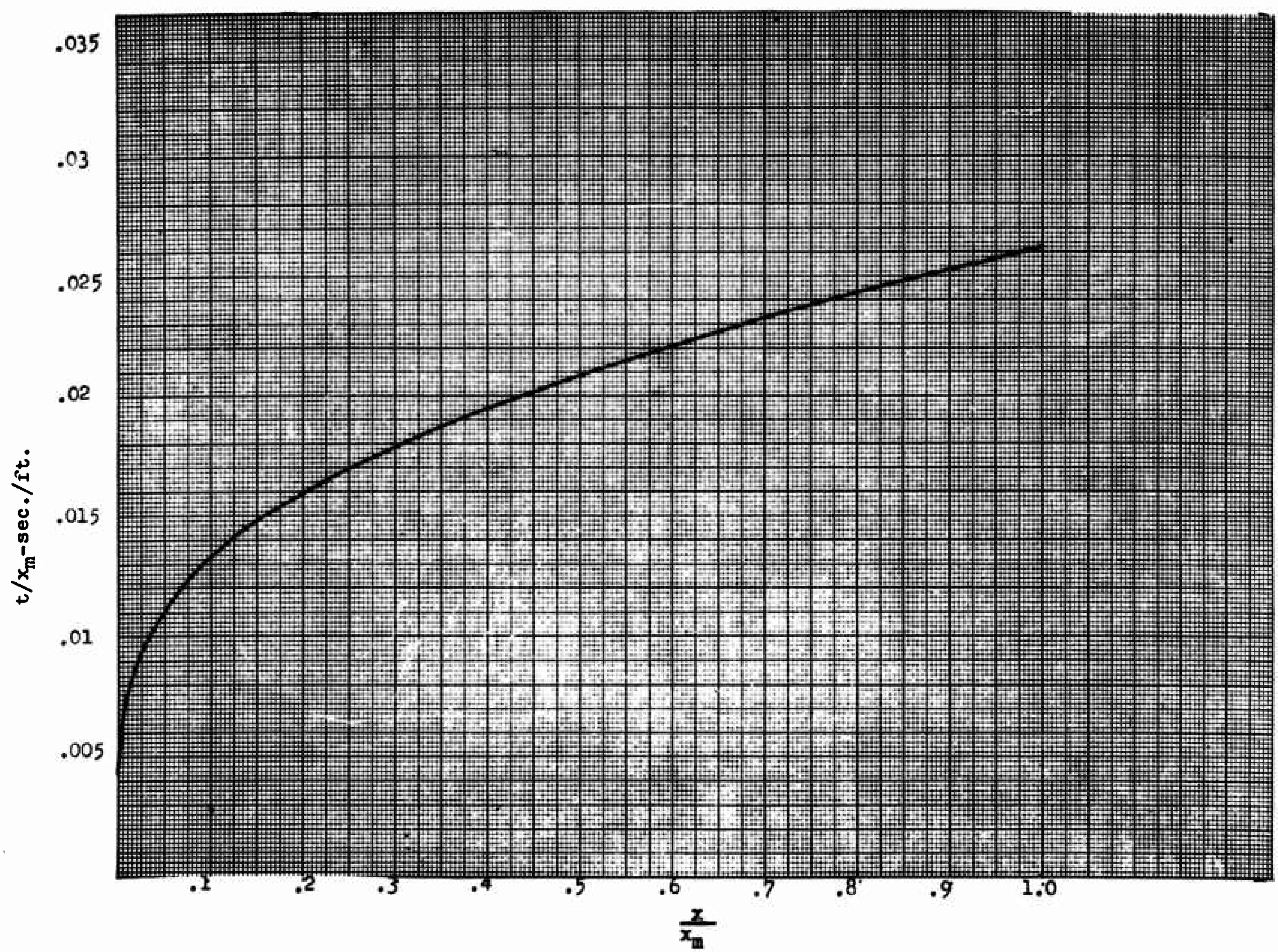


Figure 11: t/x_m Versus Displacement Fraction $\frac{x}{x_m}$, for Devices I, II, and III

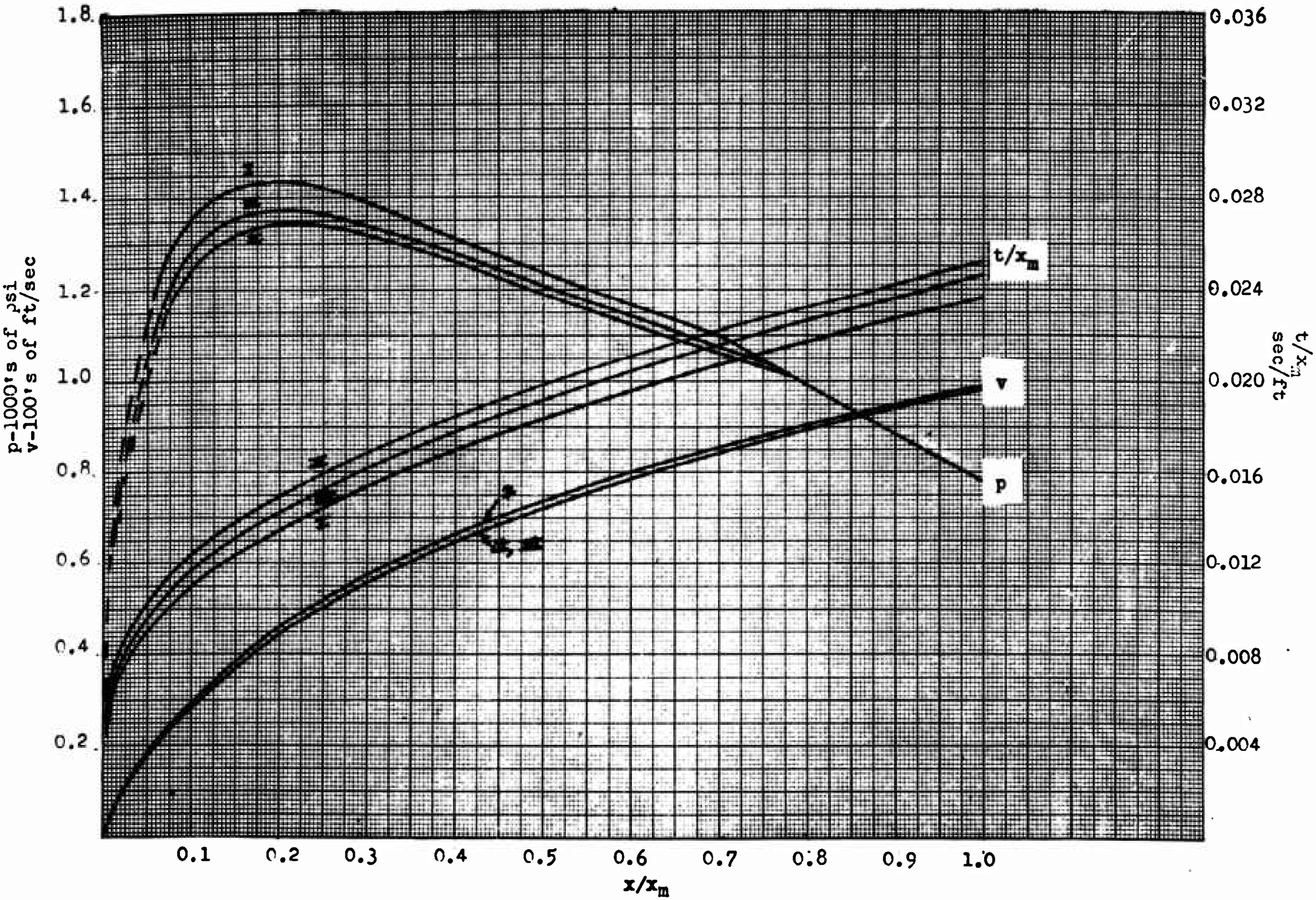


Figure 12: p, v and t/x_m Versus x/x_m for Devices I, II, III; $\theta = 90°$

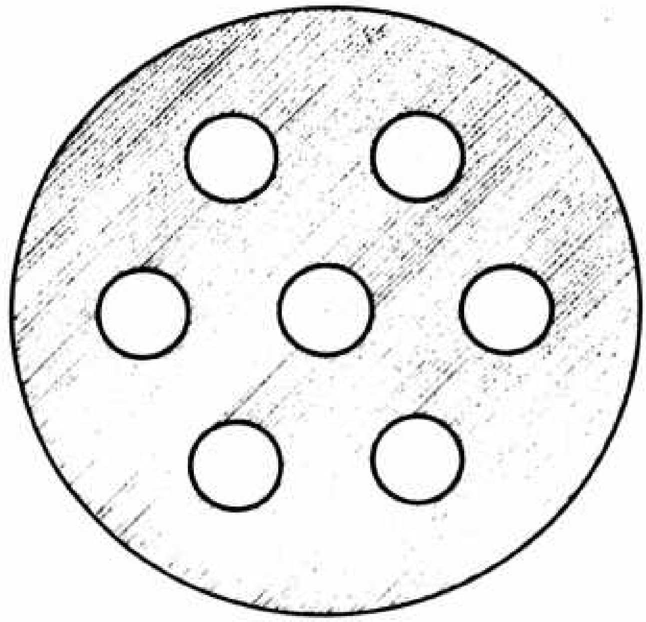


Figure 13A:
Initial Cross Section of
Seven Perf Grain

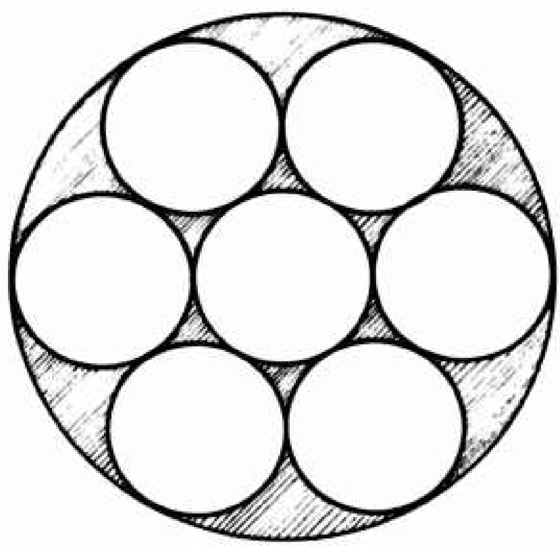


Figure 13B:
Cross Section at Splintering
of Seven Perf Grain

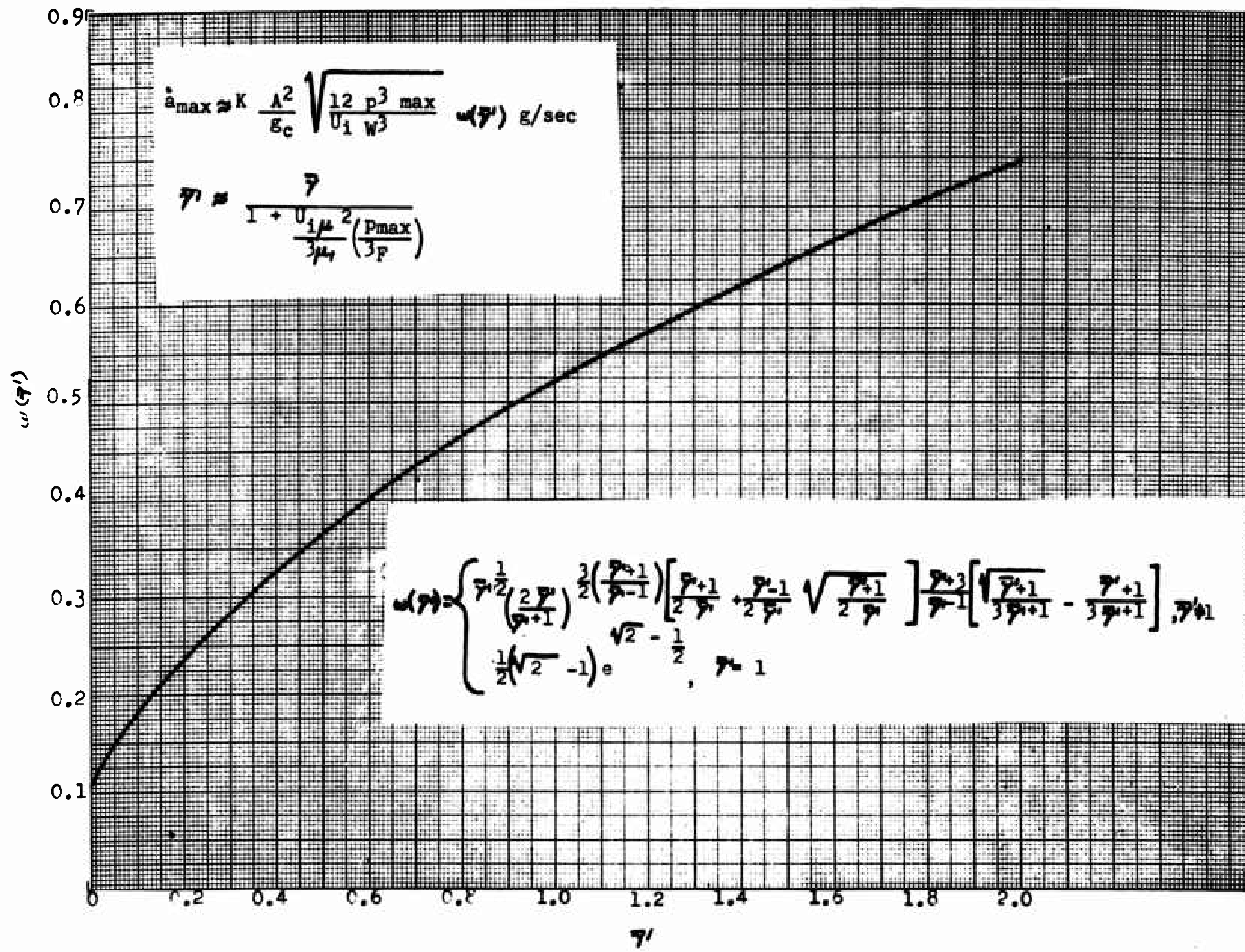


Figure 14: $\omega(\bar{\gamma}')$ Versus $\bar{\gamma}'$

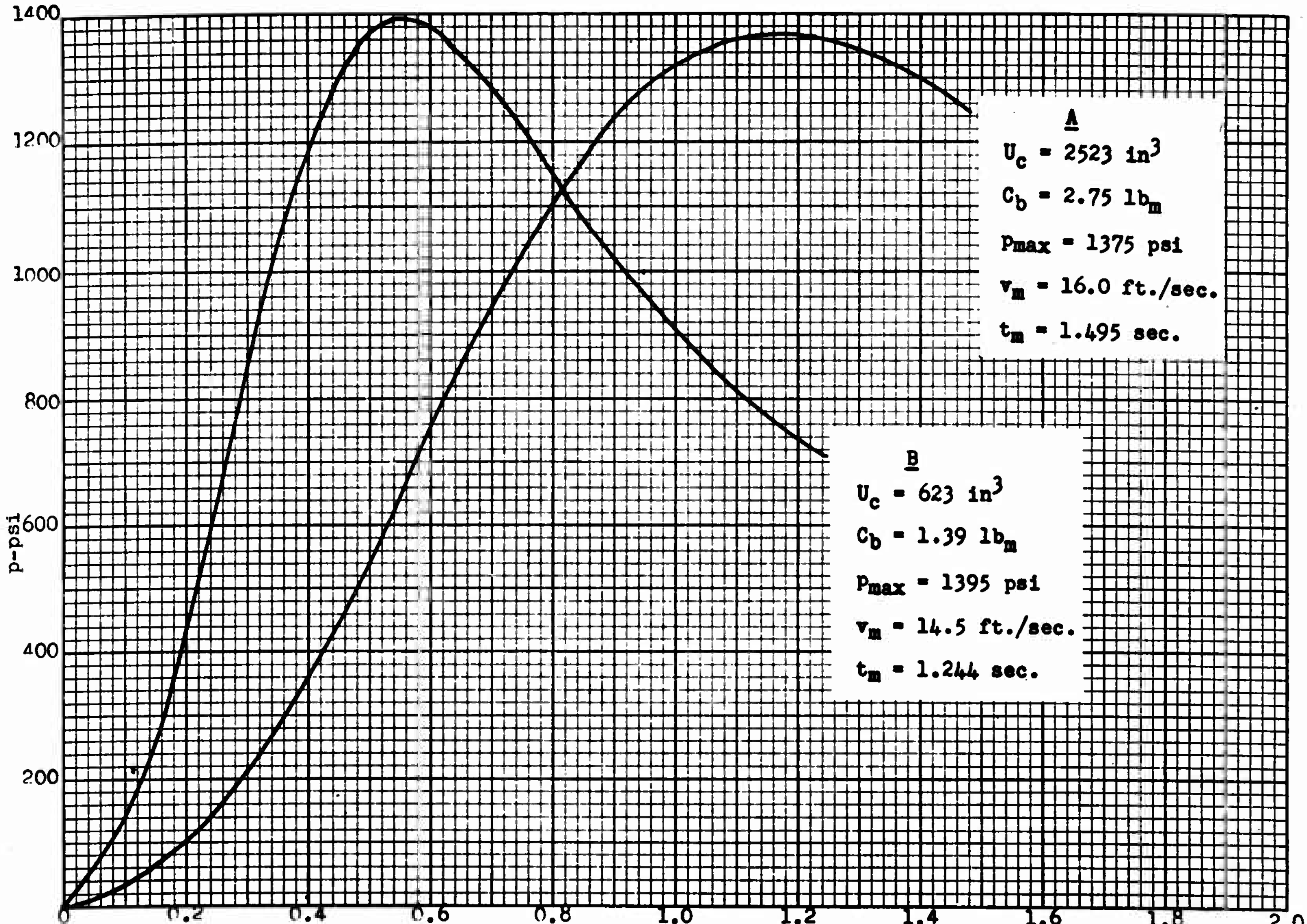


Figure 15: Pressure Versus Time for Cases A and B

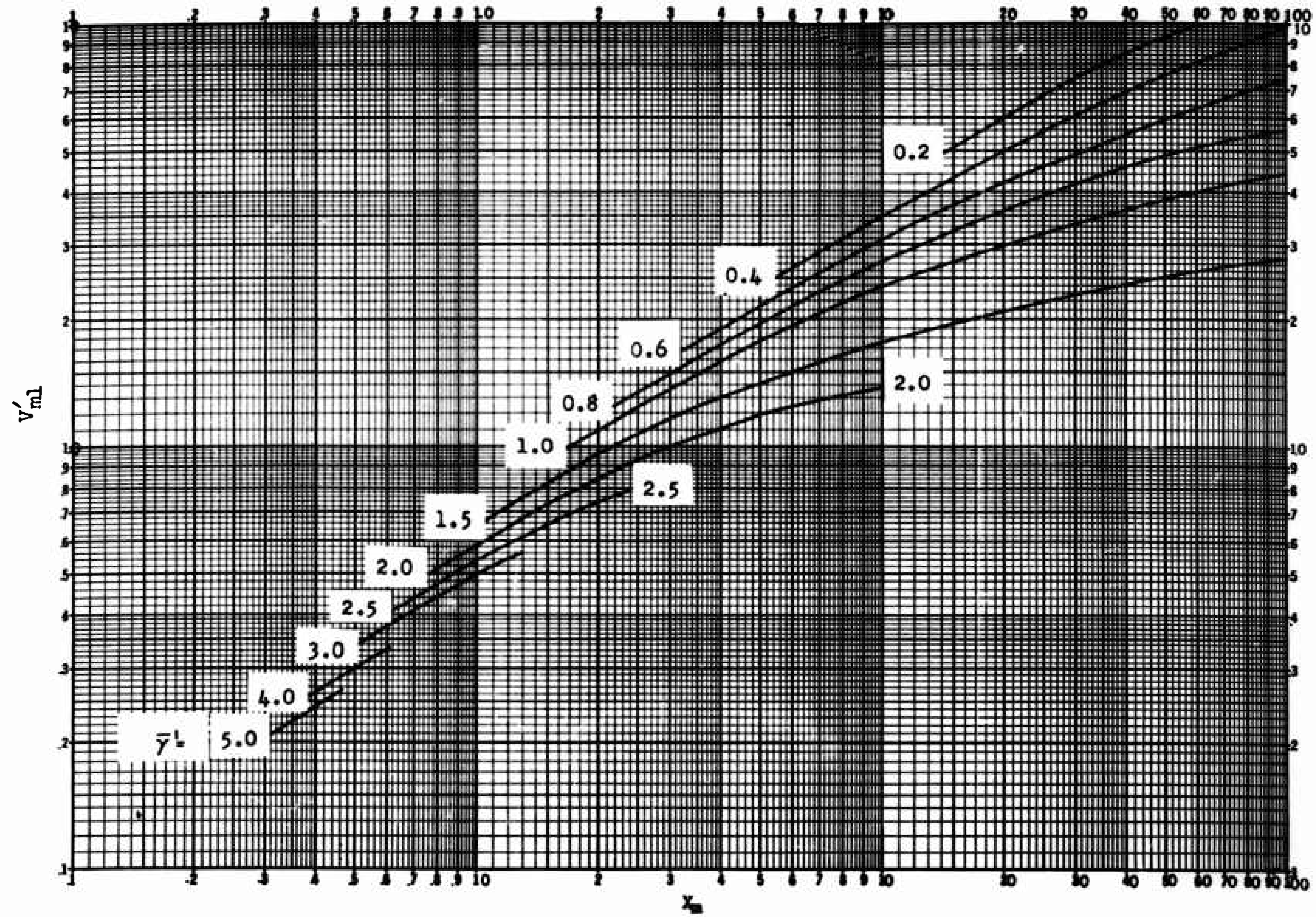


Figure 16: Solution Domain for $\bar{\gamma} = 1.295$

APPENDIX G

NOMENCLATURE

		Units
A	- cross-sectional area of piston	in^2
$\dot{a}_{max}$	- maximum rate of change of acceleration	g's/sec
B	- burning rate coefficient	$\left(\frac{in}{sec}\right)\left(\frac{in^2}{lb_f}\right)^n$
C	- mass of charge burned	lb_m
C_b	- charge weight	lb_m
C_v	- constant volume specific heat of propellant gases	$\frac{ft\text{-}lb_f}{lb_m\,°K}$
D	- diameter of propellant grain	in
d	- diameter of perforation of propellant grain	in
E	- piezometric efficiency of CAD firing $= \frac{1/2\, W v_m^2}{p_{max} A x_m}$	----
E_1	- volumetric efficiency $= \frac{1/2\, W v_m^2}{F \rho U_c}$	----
E_2	- $\frac{\widetilde{V}_m 1}{2}\left[1 - \frac{12\, A x_m}{U_c}\left(1/X_m\right)\right]$	----
$f(a, b)$	- "a function of a and b"	----
F	- impetus of propellant $= NRT_v$	$\frac{ft\text{-}lb_f}{lb_m}$
F_f	- resistance force due to friction	lb_f
g	- acceleration due to gravity	ft/sec^2
g_c	- conversion factor = 32.174	lb_m/slug
h	- length of propellant grain	in
K	- spring constant	lb_f/ft
K_1	- friction constant	----

K_2	- heat-loss constant	----
K_3	- non-horizontal firing constant	----
K_4	- neglected co-volume constant	----
k	- empirical correction factor taken to be $\frac{4}{3}$	----
k_i	- ballistic constants from Section XII	variable
L	- distance burned through propellant grain	in
L_b	- propellant web = distance burned through grain at "burnt"	in
$L_{b\,min}$	- propellant web required to attain "burnt" at end of stroke	in
N	- moles of gas per unit mass of propellant	$\frac{(gm)\ \text{moles}}{\text{gram}}$
n	- burning rate exponent	----
p	- space average gas pressure	lb_f/in^2
p_s	- pressure on base of piston	lb_f/in^2
p_{max}	- maximum pressure attained in CAD firing	lb_f/in^2
p^*_{max}	- corrected maximum pressure	lb_f/in^2
P	- dimensionless pressure variable $= \frac{U_i}{12N}\left(\frac{A}{\alpha BF\lambda}\right)^2 p$	----
P'	- dimensionless pressure variable $= \left(\frac{\lambda}{\lambda_1}\right)^2 P$	----
P_M	- mathematical maximum of P	----
P'_M	- mathematical maximum of P'	----
P_μ	- maximum value of P attained in particular CAD firing	----
P'_μ	- maximum value of P' attained in particular CAD firing	----
q_1	- ballistic parameter $= \frac{U_i}{12N}\left(\frac{A}{BF\lambda}\right)^2$	----
q_2	- ballistic parameter $= \frac{A}{BF\lambda}$	----

q_3	- ballistic parameter $= \left(\frac{A}{B}\right)^2 \frac{L_b}{F\lambda W}$	----
q_1'	- ballistic parameter $= \left(\frac{\lambda}{\lambda_1}\right)^2 q_1$	----
q_2'	- ballistic parameter $= \left(\frac{\lambda}{\lambda_1}\right) q_2$	----
q'_3	- ballistic parameter $= \left(\frac{\lambda}{\lambda_1}\right) q_3$	----
Q_1	- ballistic parameter $= q_1/\alpha^2$	----
Q_1'	- ballistic parameter $= q_1'/\alpha^2$	----
Q_2	- ballistic parameter $= q_2/\alpha$	----
Q_2'	- ballistic parameter $= q_2'/\alpha$	----
R	- universal gas constant $= 2782$	$\frac{\text{ft-lb}_f \text{ gram}}{\text{lb}_m \text{ mole } ^\circ K}$
R_{max}	- ballistic parameter $= \left(\frac{4}{2\bar{\gamma}^2 - 7\bar{\gamma} + 9}\right)^{\frac{2}{\bar{\gamma}-1}} - 1 \approx 3$	----
r	- burning rate $= \frac{dL}{dt}$	in/sec
S	- propellant burning surface	in^2
$\bar{S}$	- average burning surface $= \frac{C_b}{\rho L_b}$	in^2
S_o	- initial burning surface	in^2
S_b	- final burning surface	in^2
t	- time measured from ignition	seconds
T	- space-average propellant gas temperature	$^\circ K$
T_v	- constant-volume flame temperature of propellant gases	$^\circ K$
U_b	- barrel volume $= 12\ Ax_m$	in^3
U_c	- empty chamber volume	in^3

U_i	- initial free chamber volume $= U_c - C_b/\rho$	in^3
v	- velocity of piston	ft/sec
v_m	- ejection velocity	ft/sec
v_m^*	- corrected ejction velocity	ft/sec
V	- dimensionless velocity variable $= \frac{A}{\alpha B F \lambda} v$	----
V'	- dimensionless velocity variable $= \frac{\lambda}{\lambda_1} V$	----
V_M	- value of V at which P attains its mathematical maximum $= 1/\bar{\gamma}$	----
V'_M	- value of V' at which P' attains its mathematical maximum $= 1/\bar{\gamma}'$	----
V_{m1}	- value of V at completion of stroke if $V_b \geq V_{m1}$	----
V'_{m1}	- value of V' at completion of stroke if $V'_b \geq V'_{m1}$	----
V_b	- value of V at burnt $= \left(\frac{A}{\alpha B}\right)^2 \frac{L_b}{F\lambda W}$	----
V'_b	- value of V' at burnt $= \frac{\lambda}{\lambda_1} V_b$	----
V_{m2}	- value of V at completion of stroke if $V_b < V_{m1}$	----
V_μ	- value of V at which P attains its greatest value	----
V'_μ	- value of V' at which P' attains its greatest value	----
$\widetilde{V}_{m1}$	- the smaller root of the equation $\frac{\Gamma V_{m1}^2}{E}\lambda = \left(1 - \frac{\bar{\gamma} - 1}{2} V_{m1}\right)^{-\frac{2}{\bar{\gamma} - 1}} - 1$	----
$\breve{V}_{m1}$	- the larger root of the equation $\frac{\Gamma V_{m1}^2}{E}\lambda = \left(1 - \frac{\bar{\gamma} - 1}{2} V_{m1}\right)^{-\frac{2}{\bar{\gamma} - 1}} - 1$	----
w	- accelerated mass	lb_m

w_1	- effective accelerated mass $= (1 + K_1)\,w$	lb_m
W	- w_1/g_c	slugs
x	- displacement of piston from initial position	ft
x_m	- stroke length	ft
X	- dimensionless piston travel $= \frac{12A}{U_i}\,x$	----
X_m	- value of X at $x = x_m$	----
α	- burning rate adjustment factor	$\left(\frac{lb_f}{in^2}\right)^{n-1}$
β	- $\frac{U_c - C_b/\rho}{(C_b/L_b)^2}$	in^5/lb_m
β'	- ballistic parameter from Appendix E	----
γ	- ratio of specific heats $= 1 + \frac{NR}{C_v}$	----
$\bar{\gamma}$	- adjusted ratio of specific heats $= \left(\frac{1 + K_1 + K_2}{1 + K_1}\right)(\gamma - 1) + 1$	----
$\bar{\gamma}'$	- ballistic parameter $= \bar{\gamma} - 2\left(\frac{\lambda - \lambda_1}{\lambda_1 V_b^i}\right)$	----
Γ	- dimensionless ballistic parameter $= \frac{\bar{\gamma}}{2}\left(\frac{2\bar{\gamma}}{\bar{\gamma} + 1}\right)^{\frac{\bar{\gamma} + 1}{\bar{\gamma} - 1}}$	----
Γ'	- dimensionless ballistic parameter $= \begin{cases} \frac{\bar{\gamma}'}{2}\left(\frac{2\bar{\gamma}'}{\bar{\gamma}' + 1}\right)^{\frac{\bar{\gamma}' + 1}{\bar{\gamma}' - 1}} & \text{for } \bar{\gamma}' \neq 1 \\ \frac{e}{2} & \text{for } \bar{\gamma}' = 1 \end{cases}$	----
Δ	- loading density $= C_b/\rho U_c$	----
ϵ	- fractional increase in a ballistic parameter	----
δ	- ballistic parameter $\frac{U_i}{12\,A x_m}$	----
η	- covolume factor	in^3/lb_m

θ	- launching angle	degrees
Λ_1	- form function coefficients	lb_m/in
Λ_2		lb_m/in^2
Λ_3		lb_m/in^3
λ	- linear form function coefficient $= C_b/L_b$	lb_m/in
λ_1	- quadratic form function coefficients	lb_m/in
λ_2		lb_m/in^2
μ	- ballistic parameter $\frac{\bar{\gamma}-1}{2}$	----
μ'	- ballistic parameter from Appendix E	----
ν	- ballistic parameter $= \frac{1+X_b}{1+X_m}$	----
ν'	- ballistic parameter from Appendix E	----
ν_i	- ballistic constants from Section XII	variable
ρ	- propellant density	lb_m/in^3
σ	- ballistic constant ≈ 3	----
σ_1	- $\int_0^{\phi_{max}} B^2 \, (\alpha\phi - \phi^n)^2 \, d\phi$	lb_f/sec^2
σ_2	- $\int_0^{\phi_{max}} B \, \lvert \alpha\phi - \phi^n \rvert \, d\phi$	$\frac{lb_f}{in\text{-}sec}$
τ	- arbitrary ballistic parameter from Section XII	arbitrary
ϕ	- ballistic parameter $= \nu^{\bar{\gamma}-1}$	----
$\psi(\bar{\gamma}')$	- a function of $\bar{\gamma}'$	----
$\omega(\bar{\gamma}')$	- a function of $\bar{\gamma}'$	----
Ω	- piston diameter	in

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